Economic Model of TAC SCM Game

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Abstract

This paper presents an economic model to the problem of Supply Chain Management in Trading Agent Competition (TAC SCM). The TAC SCM marketplace is characterized by the combination of quantity competition and price competition between manufacturers(agents). We specify the quantity competition by a variation of Cournot model and view the price competition as an extension of Bertrand game. An approach of smooth-regression is introduced to copy with the non-linear fluctuation of product price by using linear price model. The results of the paper provide the solutions to the decision-making problems in TAC SCM trading agent design, including daily production, product pricing and component procuring.

1 Introduction

Trading Agent Competition (TAC) has been successfully run for several years since it was introduced in 2000 by Wellman and Wurman [10]. This annual activity offers "an international forum designed to promote and encourage high quality research into the trading agent problem". Supply Chain Management (SCM) game is a new addition to the 2003 Trading Agent Competition. The game specifies a typical three-level supply chain using a personal computer(PC) manufacturing scenario[1]. In the chain, three groups of entities: *component suppliers*, *PC assemblers*, and *end customers* are linked by two marketplaces: *component market* and *PC product market*. Participants of the game are required to design an automated trading agent, acting as a PC assembler, capable of competing with other agents for customer orders, negotiating with suppliers for components and managing daily assembly activities. Building such an agent, many highly sophisticated technologies are required:

- Online bidding in non-standard multiple-object auctions;
- Automated multiple-item negotiation;
- Supply chain formation and integration;
- Adversary and business partner modelling;
- Production scheduling under highly dynamic market situation;
- Computation- and communication-effective solutions to network bottleneck.

This paper deals with the strategic issues of TAC SCM agent design. We focus on the problems of *daily production, product pricing* and *market-clearing price prediction*, which are known as the most important issues involved in TAC SCM agent design. By differentiating the quantity competition and price competition in TAC SCM product market, we locate the solution of daily production in a variation of a Cournot model and product pricing problem in an extension of Bertrand game[6]. To cope with the non-linear fluctuation of product price, we introduce an approach of smooth-regression to reduce the non-linear factors of market demands on the market-clearing price. All the solutions to the problems have been implemented and tested in our agent, named "jackaroo", which was one of the best agents among TAC-03 participants[1]. The detail strategies used in the agent have been presented in [13].

2 TAC SCM product market

It was shown in TAC-03 that the main challenge in SCM trading agent design was the decision-making of daily production and product pricing. The TAC SCM product market involves both quantity competition and price competition. Quantity competition determines the product supply of the market while price competition reflects the demand.

[1]jackaroo received the third place in the qualifying round, the first in the seeding round 1 and the fourth in the seeding round 2. Unfortunately we were unable to proceed with the final round due to network problems at the conference venue.

2.1 Modelling Quantity Competition

The TAC SCM product market is a typical oligopoly where the manufacturers (agents) can choose quantities of products to supply within their production capacity. The market price of each product replies on the aggregate quantity of all manufacturers. Since each manufacturer's pay-off structure is common knowledge to each agent, the quantity competition of the market can be easily specified with Cournot model with a slight variation[9]. To maximize the use of the traditional microeconomic theories, we temporally assume that all PC products are homogeneous with the same market price and the same production costs for each manufacturer. In the later sections, we will release some of the assumptions.

2.1.1 Basic model

In general, we assume that there are n manufactures competing the market. Let q_i denote the quantity of PC produced by manufacturer i and Q the aggregate quantity on the market, that is, $Q = \sum_{i=1}^{n} q_i$. Assume that the market demand (expressed by customer's request for quotes(RFQs)) for all products is D. If the aggregate quantity on the market is no more than the market demand, we assume that the market-clearing price of the product is constant at p_0[2]. If the aggregate quantity is larger than the market demand, the market-clearing price is decreasing with over level of products on the market until the price drops to 0. Let $P(Q)$ denote the market-clearing price over aggregate quantity Q. Then

$$P(Q) = \begin{cases} p_0, & \text{if } Q \le D; \\ \Gamma(Q), & \text{otherwise.} \end{cases}$$

where $\Gamma(Q)$ is a monotonous decreasing function, called *product depreciation function*. Specially, if the price decline is linear, the price function can be further simplified:

$$P(Q) = \begin{cases} p_0, & \text{if } Q \le D; \\ p_0 - \gamma(Q - D), & \text{if } D < Q \le D + \frac{p_0}{\gamma}; \\ 0, & \text{otherwise.} \end{cases}$$

where γ is called the *depreciation coefficient* ($\gamma > 0$).

Let δ be the marginal cost of the product and c_0 be the fixed cost of each manufacturer. The cost for each manufacturer to produce q products is then:

$C(q) = \delta q + c_0$, where δ and c_0 are non-negative.

A *strategy*, q_i, of manufacturer i is the quantity the manufacturer chooses to produce. A *strategy profile*, S, is a decision of production by all manufactories: $(q_1, \cdots, q_n)$, where $q_i \ge 0$ for any i.

The profit of each manufacturer i (payoff function) can then be written as:

$\pi_i(S) = q_i P(Q) - C(q_i) = q_i P(Q) - \delta q_i - c_0$

If we assume that the price function is linear, the payoff function can be further specified as:

$$\pi_i(S) = \begin{cases} (p_0 - \delta)q_i - c_0, & \text{if } Q \le D; \\ (p_0 - \delta)q_i - \gamma q_i(Q - D) - c_0, & \text{if } D < Q \le D + \frac{p_0}{\gamma}; \\ -\delta q_i - c_0, & \text{otherwise.} \end{cases}$$

2.1.2 Nash equilibrium

Now the strategy of daily production to each manufacturer can be stated as the following profit-maximization problem: *for each player i, q_i^* solves the optimization problem:*

$$\max_{0 \le q_i < \infty} \pi_i(q_1^*, \cdots, q_{i-1}^*, q_i, q_{i+1}^*, q_n^*)$$

It is well-known that such a strategy profile $(q_1^*, \cdots, q_n^*)$ is called *a Nash equilibrium*.

Based on the linear assumption of price function, we have the following result.

Theorem 1 *Assume that the product depreciation function Γ is linear. If $p_0 > \delta$, there exists a unique Nash equilibrium $(q_1^*, \cdots, q_n^*)$ to the problem where if $D \ge n\frac{p_0-\delta}{\gamma}$, $q_i^* = \frac{1}{n}D$; if $D < n\frac{p_0-\delta}{\gamma}$, $q_i^* = \frac{1}{n+1}(D + \frac{p_0-\delta}{\gamma})$ for each i.*

Example 1 *Consider a PC marketplace with n suppliers. Assume that $p_0 = 1800$, $\gamma = 0.25$, $\delta = 1600$, $c_0 = 0$ and $D = 2000$. Then the price function is*

$$P(Q) = \begin{cases} 1800, & \text{if } Q \le 2000; \\ 2300 - 0.25Q, & \text{if } 2000 < Q \le 9200; \\ 0, & \text{otherwise.} \end{cases}$$

For each manufacturer i, its profit is decided by the following function:

$$\pi_i(S) = \begin{cases} 200q_i, & \text{if } q_i \le 2000 - \sum_{k \ne i} q_k; \\ -0.25q_i^2 + 200q_i - 0.25q_i \sum_{k \ne i} q_k, & \text{if } 2000 - \sum_{k \ne i} q_k < q_i \le 9200 - \sum_{k \ne i} q_k; \\ -1600q_i, & \text{otherwise.} \end{cases}$$

The following table lists the equilibrium production in case of no more than 6 manufacturers.

Number of Manufacturers	*Equilibrium Production*	*Aggregate Production*	*Market Price*
1	*2000*	*2000*	*1800*
2	*1000*	*2000*	*1800*
3	*700*	*2100*	*1775*
4	*560*	*2240*	*1740*
5	*467*	*2333*	*1717*
6	*400*	*2400*	*1700*

[2] If market demand is more than supply, agents can normally get customer orders with reserve price or a little bit less.

The last line of the table shows that if the market situation follows the setting of the example, a TAC SCM agent should maximize its use of assembly line capacity, which coincides the real situation in TAC-03[3].

Corollary 1 *Let S^* is the Nash equilibrium in Theorem 1, then for any $i \leq n$,*
$\pi_i(S^*) = \frac{\gamma}{(n+1)^2}(D + \frac{p_0-\delta}{\gamma})^2 - c_0$.

The result shows that changing of component price have a quadratic effect on agent profit. This explained that reducing component cost was one of the dominant strategies in TAC-03 SCM game.

2.1.3 Varying customer demands

It is easy to see that our model above is different from the classical Cournot model. Since market prices of each product is capped with customer reserve prices, the manufactures cannot fully control the market with their production. The variation results in the difference of Nash equilibrium: *if market demand is so big that a small reduction of price would significantly result a big drop of manufacturer's profit, no manufacturer has the incentive to produce more than its portion of market.* This can be illustrated by the following example.

Example 2 *Consider a duopoly where two manufacturers compete a market of one PC product. Assume that $p_0 = 2000$, $\gamma = 0.5$, $\delta = 1500$, $c_0 = 0$.*

Suppose that the current market demand $D = 3000$. Then $D > n\frac{p_0-\delta}{\gamma}$. According to Theorem 1, the equilibrium quantity for each manufacture is 1500, which coincides the amount of its market portion. Therefore no one has the incentive to produce more than it should do. Figure 1 depicts the payoff curve in this situation where the peak of payoff function of a manufacturer is located at exact its market portion (1500).

Figure 2 illustrates the situation of small market demand where $D = 1000$, which is less than $n\frac{p_0-\delta}{\gamma}$. It can be seen that a manufacturer would benefit from an extra production of 167.

We remark that in the real TAC SCM game, the depreciation coefficient γ could vary with market demands. In section 3.3 we will derive the correlation between the depreciation coefficient and market demand through a statistic analysis on TAC-03 SCM game.

[3]According to the TAC-03 SCM specification, the production capacity of each PC manufacturer is 2000 cycles per day which allows each manufacturer to produce up to 400 PCs per day(364 on average)

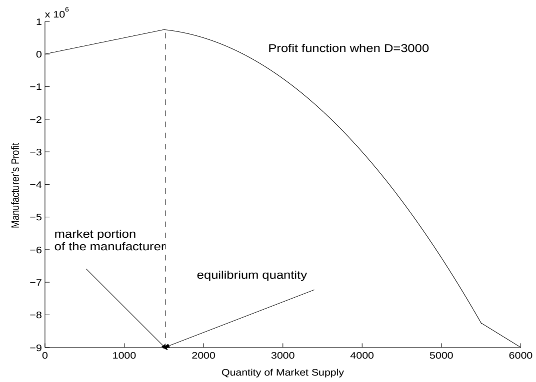

Figure 1. Equilibrium under large market demands

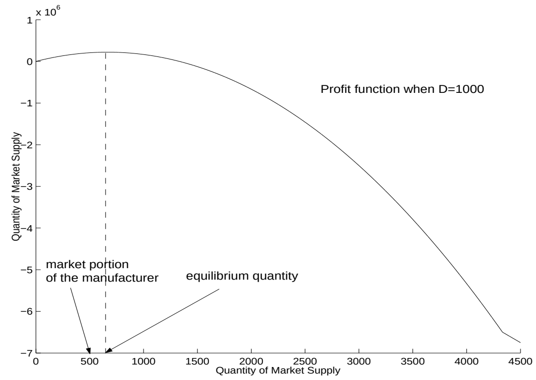

Figure 2. Equilibrium under small market demands

2.2 Modelling Price Competition

In last section, we make the decision of daily production based on a comprehensive perspective on the market. In other words, we assumed that all products in the market are homogenous with uniform price for every product and every agent. This assumption is a simplification on the averaging base. Additionally we did not consider the limitation of production capacity on each agent. In the real TAC SCM game, however, prices of products result from a first-price sealed-bid auction. Agents bid for customer orders with their own prices. The quantity an agent supplies is limited by its nominal capacity of production and its component supply[1].

Once the production has been decided, the produced products will be put on to the market to participate the price competition. Economists have found that price competition is normally more severe than quantity competition[6]. In an oligopoly where more than one firms compete in price, the market equilibrium would lead to a case that all firms earn zero profits. This extreme case is known as the Bertrand paradox[4]. Fortunately, the TAC SCM product market does not fit in the simple Bertrand model of price competition since the market contains multiple product selling and production capacity of each manufacturer is limited. Additionally, production costs of each manufacturer are different. Due to the complexity of these features, the analysis of price competition becomes much harder.

To simplify the exploration, we still keep the assumption of homogenous products but release the assumption of uniform prices and production costs. Formally, let q_i be the scheduled daily production of agent i and δ_i the marginal cost of the products. Assume that o_i is the totally ordered amount the agent received on a day at an average price of p_i (different orders could have different prices). Additionally, we assume that η_i is the average inventory cost of the product. Note that this cost is not necessarily the real inventory carrying cost but comprises the cost of unsold products at the end of the game, the revenue of the products when they are sold in the future (represented in negative value) and bank interests[4]. Therefore the value of η_i could be negative if the revenue can eventually offset the cost. Using these assumption, the one day profit of agent i will be:

$$\pi_i = o_i p_i - \delta_i q_i - \eta_i(q_i - o_i) \tag{1}$$

Putting this aside, we consider how much an *average agent* can earn in one day. Again we use D to denote the customer demand for the product per day. Let p and δ be the average market price and average component cost, respectively. Then an average agent can earn on the day:

$$\pi_{average} = (p - \delta)\frac{D}{n} \tag{2}$$

where n is the total number of agents.

If agent i expects her profit to be no worse than an average, i.e.,

$$o_i p_i - \delta_i q_i - \eta_i(q_i - o_i) \geq (p - \delta)\frac{D}{n}$$

then the total ordered amount she receives should satisfy:

$$o_i \geq \frac{(p-\delta)\frac{D}{n} + (\delta_i + \eta_i)q_i}{p_i + \eta_i} \tag{3}$$

If the actual order an agent receives is less than she expects (does not satisfies the condition (3)), she will need to lower her bidding price; otherwise she should keep or raise her product price.

On the other hand, the condition (3) also provides a solution to product pricing. If we view o_i as the average ordered amount an agent receives, then the agent should set her product price as

$$p_i = \frac{(p-\delta)\frac{D}{n} + (\delta_i + \eta_i)q_i}{o_i} - \eta_i \tag{4}$$

We remark that the actual pricing algorithm we used in our agent is much more complicated than simply calculating the above expression. Application of this expression relies on the estimation on the *current market price*, *component cost* and the *"inventory cost"*. In the next section 3.2, we will switch to the forecasting of market product price.

3 Market price forecasting

We have seen that market price prediction plays extremely important role in product pricing. Observations on the last SCM game showed that there were many factors which affected the market prices of PC products. Besides customer demands and market product supply, agent's individual pricing strategies, stages of a game, component supply also played very import roles in the product market. In this section, we investigate relationship between market demand and product price, and use the relation as the way to predict market-clearing price.

3.1 Correlation between market supply and market-clearing price

One assumption we have made in our product market model is that there would be a linear relationship between market supply and market-clearing price. To verify this assumption, we conducted a statistic analysis on the TAC-03 SCM game. The data we used comes from the TAC-03 SCM final last round, total 16 games on both tac5 and tac6 servers. The reason we chose these games is that all these games played by exactly the same players. So we can minimize the effect of pricing strategies used by different players. For each game, we picked up all negotiation data for all days of a game, including customer RFQ quantities, offered product quantities, customer's reserve prices and market-clearing prices of all PC products. These data were classified and averaged according to the ranges of market demands (see Table 1). The coefficients of correlation between market supply and depreciation of market price (the gap between customer reserve price and market-clearing price) were then calculated. Finally we averaged all the data of 16 games. Table 1 shows the statistic result.

This table clearly shows a significant linear correlation between quantities of market supply and depreciations of market price in the *middle range* of market demand. However, the relation is getting weaker at the lower end and the higher end of market demand. The reason is the following. When market demand is low, the competition of the

[4] In fact, in TAC-03 SCM specification, the inventory carrying costs for both PC products and components are assumed to be zero.

Market Demand	Average Market Demand	Average Market Supply	Average Reserve Price	Average Market Price	Correlation Coefficient
≤ 800	728	2313	2003	957	0.53
800-1200	957	2802	2019	1093	0.55
1200-1600	1395	2178	2098	1622	0.83
1600-2000	1782	2040	2117	1807	0.81
2000-2400	2202	1325	2177	1988	0.68
2400-2800	2597	1631	2176	2006	0.52
2800-3200	3040	2027	2177	1982	0.24
> 3200	3440	2360	2199	2000	0.20

* *The result is based on TAC-03 SCM Final game 1264-1271@tac5 and 1423-1430@tac6.*

Table 1. Correlation between market-clearing price and market supply in product market.

Market Demand	b_0	b_1	Actual Market Price	Forecast	Forecast Precision %
≤ 800	441.6	0.25	957	1380	55.8
800-1200	325.7	0.21	1093	1488	63.9
1200-1600	115.5	0.18	1622	1735	93.0
1600-2000	95.8	0.11	1807	1827	98.9
2000-2400	90.7	0.16	1988	1729	87.0
2400-2800	122.3	0.15	2006	1666	83.1
2800-3200	161.7	0.04	1982	1886	95.2
> 3200	161.1	0.01	2000	1996	99.8

* *The result is based on TAC-03 SCM Final game 1264-1271@tac5 and 1423-1430@tac6.*

Table 2. Market price forecasting by using Linear Regression

product market becomes much severe. In this case, agents would apply an non-linear price-cutting strategy to compete customer orders. On the other hand, when market demand exceeds market supply, the market-clearing price will approach to the customer reserve price. The product prices are then mostly determined by agents' pricing strategies rather than the quantity of market supply.

3.2 Smooth-regression for price forecasting

We have seen that the price function of product market may not simply be approximated by a linear function. This means that if we use a linear forecasting approach to predict market price, the result could be poor.

Table 2 shows the result of prediction by using linear regression on the data as before (TAC-03 SCM Final). The column 2 and 3 in the table give the sample estimate of regression coefficients β_0 and β_1 under different market demands (averaged values with 16 games). Column 4 and 5 show the actual average market-clearing prices and their forecast values. Column 6 shows the precisions of forecasting. Note that in the calculation of regression coefficients, the dependent variable we used is not the market-clearing price but the price difference between reserve price and market-clearing price (price depreciation). Therefore the linear regression line gives us only an estimate of market price depreciation. We need to recover the market-clearing prices by using the average reserve prices:

$$p_{forecast} = p_{reserve} - (b_0 + b_1 Q) \qquad (5)$$

where $p_{reserve}$ and Q are the average reserve price and average quantity of market supply, respectively. $p_{forecast}$ is the forecast value of market-clearing price.

Observing Table 2, we found that the forecasting results for the high range of market demands is not as bad as we expected. The reason for this is that the actual market prices in this range uniformly distribute around the regression line(this can be observed from a scatter diagram of the raw data). Therefore the average values of market-clearing price match the forecast values very well. Nevertheless, the quality of prediction in the low range of market demands is quite poor. To solve the problem, instead of introducing a non-linear forecasting model, we adjust the regression parameters so that the non-linear features can be recovered from the trend of parameter variations. We call this approach *smooth-regression*.

First we calculate the estimate values of regression parameter b_0 and b_1 against market demands on previous game data (see Table 2 column 2 and 3) by using simple linear regression model. Next we find the best fitting curve to match each regression parameter. The Figure 3 and Figure 4 depicts the respective fitting curves of the regression parameters b_0 and b_1 varying with market demands. The sample data come from column 2 and 3 of Table 2.

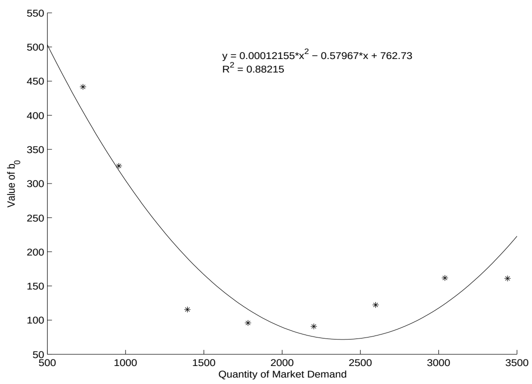

Figure 3. Fitting curve of b_0 over market demand.

Figure 3 shows that the trend of β_0 over market demand fits a quadratic curve. It results in a fitting function as follows:

$$\hat{b}_0 = 0.00012155 * D^2 - 0.57967 * D + 762.73 \quad (6)$$

However, the trend of β_0 fits very well a straight line with the norm of residuals as small as 0.08725:

$$\hat{b}_1 = -0.000076109 * D + 0.29303 \quad (7)$$

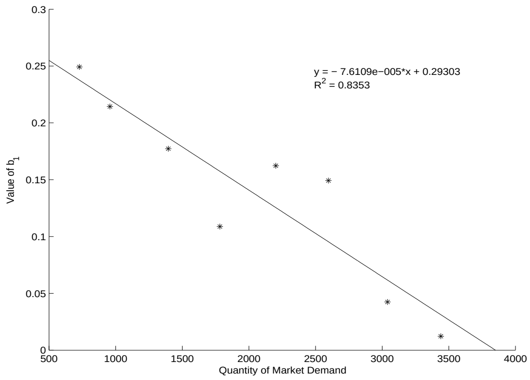

Figure 4. Fitting curve of b_1 over market demand.

With these functions, we can easily yield the estimate values of the linear regression coefficients. Luckily, *this fitting process smoothed the stochastic factors from agents' pricing strategies*. Table 3 shows the result of market price prediction by using the smoothed regression parameters, where the values of $\hat{b}_0$ and $\hat{b}_1$ were calculated by function (6) and (7). We can see that the regression parameters in the lower end of market demand have been successfully adjusted by the smoothing process. The forecasting precision becomes to be more irrelevant to the market demand. The overall forecasting quality has also been significantly improved.

Market Demand	$\hat{b}_0$	$\hat{b}_1$	Actual Market Price	Forecast Price	Forecast Precision %
$<= 800$	405.1	0.24	957	1048	91.0
800-1200	319.3	0.22	1093	1083	99.1
1200-1600	190.6	0.19	1622	1500	92.1
1600-2000	115.7	0.16	1807	1680	92.7
2000-2400	75.7	0.13	1988	1935	97.0
2400-2800	77.1	0.10	2006	1943	96.5
2800-3200	123.8	0.06	1982	1928	97.5
> 3200	207.0	0.03	2000	1918	96.1

* *The result is based on TAC-03 SCM Final game 1264-1271@tac5 and 1423-1430@tac6.*

Table 3. Market price forecasting by using smooth regression.

We have shown that with smooth regression, we can keep the linear assumption for the purpose of market price forecasting. You may argue that the forecasting results shown in Table 3 does not represent the general online quantity of prediction since all the parameters were yielded via offline learning. To show whether the smoothing functions can be used in online price forecasting, we picked up twenty games from TAC-03 SCM seeding round 2: game 628-637 at tac5 and game 778-787 at tac6 and analyzes the forecasting results. Table 4 shows a summery of the statistical results. Note that the estimate values of b_0 and b_1 are calculated by the smoothing functions (6) and (7) (we use the average daily market demands as the values of independent variable). The forecast values are given by Equation (5) (we omitted the average reserve prices from the table). The result shows that the smooth regression produces a reasonable high quality online price forecasting. In our agent implementation, the parameter can be dynamically adjusted through a feedback mechanism. The detail algorithm will be shown in a separate paper.

Market Demand	$\hat{b}_0$	$\hat{b}_1$	Actual Market Price	Forecast Price	Forecast Precision %
≤ 800	402.8	0.2372	1209	1039	86.0
800-1200	317.6	0.2198	1339	1074	80.2
1200-1600	191.4	0.1871	1691	1362	80.5
1600-2000	115.2	0.1571	1782	1518	85.2
2000-2400	77.0	0.1275	1834	1696	92.5
2400-2800	76.8	0.0958	1934	1845	95.4
2800-3200	123.6	0.0618	1905	1831	96.1
> 3200	206.3	0.0314	1937	1837	94.9

* *The result is based on TAC-03 SCM Seeding round game 628-637 @tac5 and 778-787@tac6.*

Table 4. Market price forecasting by using smooth regression (continue).

3.3 Estimation of price parameters

Let's come back to our product market model. As we have seen that our solution to daily production is based on the linear assumption of price function. We also shown that this assumption is not true when market demand is extremely low or high. By using the smooth regression function given by (6) and (7), we can adjust the price function to reduce the non-linear affects of market demand.

Let Q be the aggregate quantity of the products and $\hat{P}(Q)$ the estimated value of market-clearing price. According to equation 5,

$\hat{P}(Q) = p_{reserve} - (\hat{b}_0 + \hat{b}_1 Q)$

Since $P(D) = p_0$, by using (6) and (7) we have,

$$\begin{aligned}\hat{p}_0 &= \hat{P}(D)\\ &= p_{reserve} - (\hat{b}_0 + \hat{b}_1 D)\\ &= p_{reserve} - 0.000045441D^2 + 0.28664D\\ &\quad -762.73\end{aligned}$$

Similarly the estimate for the depreciation coefficient is given by:

$\hat{\gamma} = \hat{b}_1 = -0.000076109D + 0.29303.$

Once the price function is ready, it is easy to decide daily production. For instance, suppose that the average customer reserve price is $p_{reserve} = 2000$, then

$\hat{p}_0 = -0.000045441D^2 + 0.28664D + 1237.27.$

$\hat{\gamma} = -0.000076109D + 0.29303.$

Assume that the average marginal cost of PC products is $\delta = 1378$.[5] According to Theorem 1, an agent should fully use its production capacity once the market demand is more than 1450 PC per day.

4 Conclusion

This paper presented a theoretical model for TAC SCM game. This model provided the solutions to several key components in TAC SCM trading agent design, including decision-making of daily production, product pricing and market price forecasting.

TAC SCM game is representative of a broad range of supply chain situation. It is not hard to apply the game structure to other domains. Additionally, The game not only provides a competitive environment to evaluate different trading strategies and different structure of agents but can also act as a testbed for examining artificial e-market rules. It is no doubt that the models and approaches introduced here can also be applied to the modelling of other e-trading systems.

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Proofs of Theorems

Proof of Theorem 1: Let $Q = \sum_{i=1}^{n} q_i$. According to the linear assumption, the payoff function can be rewritten as follows:

$$\pi_i(S) = \begin{cases} (p_0-\delta)q_i - c_0, & \text{if } q_i \le D - \sum_{k\neq i} q_k;\\ -\gamma q_i^2 + (p_0 - \delta - \gamma\sum_{k\neq i} q_k + \gamma D)q_i - c_0, & \\ & \text{if } D - \sum_{k\neq i} q_k < q_i \le D + \frac{p_0}{\gamma} - \sum_{k\neq i} q_k;\\ -\delta q_i - c_0, & \text{otherwise.}\end{cases}$$

Therefore we have

[5] This value is the average cost of PC product calculated based on the TAC-03 SCM final game data:

$$\delta = \frac{total\ material\ cost}{total\ PC\ sold}$$

Therefore the "inventory cost" has been considered .

$\frac{\partial \pi_i}{\partial q_i} =$

$$\begin{cases} p_0 - \delta, & \text{if } q_i \le D - \sum_{k\neq i} q_k; \\ -2\gamma q_i + p_0 - \delta - \gamma \sum_{k\neq i} q_k + \gamma D, & \\ & \text{if } D - \sum_{k\neq i} q_k < q_i \le D + \frac{p_0}{\gamma} - \sum_{k\neq i} q_k; \\ -\delta, & \text{otherwise.} \end{cases}$$

First we consider the case that $D > \frac{n}{n+1}\frac{p_0-\delta}{\gamma}$. We prove that $S^* = (q_1^*, \cdots, q_n^*)$ is a Nash equilibrium if $q_i^* = \frac{1}{n}D$.

For any i, let $S = (q_1^*, \cdots, q_i^*, q_i, q_i^*, \cdots, q_n^*)$. If $q_i < D - \sum_{k\neq i} q_k^*$, then

$$\begin{aligned} \pi_i(S) &= (p_0-\delta)q_i - c_0 \\ &< (p_0-\delta)(D - \sum_{k\neq i} q_k^*) - c_0 \\ &= (p_0-\delta)(D - \tfrac{n-1}{n}D) - c_0 \\ &= (p_0-\delta)(\tfrac{1}{n}D) - c_0 \\ &= \pi_i(S^*) \end{aligned}$$

If $D - \sum_{k\neq i} q_k^* < q_i \le D + \frac{p_0}{\gamma} - \sum_{k\neq i} q_k^*$, then

$$\begin{aligned} \frac{\partial \pi_i}{\partial q_i} &= -2\gamma q_i + p_0 - \delta - \gamma \sum_{k\neq i} q_k^* + \gamma D \\ &= -2\gamma q_i + p_0 - \delta - \tfrac{n-1}{n}\gamma D + \gamma D \\ &< -2\gamma(D - \sum_{k\neq i} q_k^*) + p_0 - \delta + \tfrac{1}{n}\gamma D \\ &= -2\gamma(D - \tfrac{n-1}{n}D) + p_0 - \delta + \tfrac{1}{n}\gamma D \\ &= -\tfrac{2}{n}\gamma D + p_0 - \delta + \tfrac{1}{n}\gamma D \\ &= -\tfrac{1}{n}\gamma D + p_0 - \delta \end{aligned}$$

Since $D \ge n\frac{p_0-\delta}{\gamma}$, we obtain that $\frac{\partial \pi_i}{\partial q_i} \le 0$. Thus $\pi_i(S^*) \ge \pi_i(S)$.

If $q_i \ge D + \frac{p_0}{\gamma} - \sum_{k\neq i} q_k^*$, then $q_i \ge q_i^*$. It follows that $\pi_i(S) = -\delta q_i - c_0 \le -\delta q_i^* - c_0 \le (p_0-\delta)q_i^* - c_0 = \pi_i(S^*)$. So we have proved that S^* is a Nash equilibrium.

Next we consider the case when $D \le n\frac{p_0-\delta}{\gamma}$. Since the derivation of the function $-\gamma q_i^2 + (p_0 - \delta - \gamma\sum_{k\neq i} q_k^* + \gamma D)q_i - c_0$ at point $D - \sum_{k\neq i} q_k^*$ is non-negative, it is easily to see that the maximum of the payoff function $\phi_i(S)$ lies in the interval $[D - \sum_{k\neq i} q_k^*, D + \frac{p_0}{\gamma} - \sum_{k\neq i} q_k^*]$. The Nash equilibrium of the problem is then the solution of the following set of linear equations (first-order conditions):

$$q_i^* + \frac{1}{2}\sum_{k\neq i} q_k^* = \frac{1}{2}D + \frac{(p_0-\delta)}{2\gamma}, \ \ i = 1, \cdots, n, \qquad (8)$$

solving the set of equations yields

$q_i^* = \frac{1}{n+1}(D + \frac{p_0-\delta}{\gamma})$ for each i.

Now we prove the uniqueness. First we prove that a strategy profile $S^* = (q_1^*, \cdots, q_n^*)$ is a Nash equilibrium only if it satisfies the following condition:

$$q_i^* \ge D - \sum_{k\neq i} q_k^*, \text{ for all } i \le n. \qquad (9)$$

Suppose that there exist i_0 such that $q_{i_0}^* < D - \sum_{k\neq i} q_k^*$. Let $q_{i_0} = D - \sum_{k\neq i} q_k^*$. Since $p_0 - \delta > 0$, it is obvious that $\pi(S^*) < \pi(q_1^*, \cdots, q_{i_0-1}^*, q_{i_0}, q_{i_0+1}^*, \cdots, q_n^*)$. Therefore S^* is not a Nash equilibrium. According to equation 9, if there is a j such that $q_j^* \ge D - \sum_{k\neq i} q_k^*$, then for all i, $q_i^* = \frac{1}{n}D$. Otherwise, S^* must satisfies the first-order condition 8. Therefore $q_i^* = \frac{1}{n+1}(D + \frac{p_0-\delta}{\gamma})$ for each i. ¶