A Logical Multi-Demand Bargaining Model with Integrity Constraints

Xiaoxin Jing[a], Dongmo Zhang[b,*], Xudong Luo[a,**], Jieyu Zhan[a]

[a]*Institute of Logic and Cognition, Sun Yat-sen University, China.*
[b]*Intelligent System Lab, University of Western Sydney, Australia.*

Abstract

This paper proposes a logical model of multi-demand bargaining with integrity constraints. We also construct a simultaneous concession solution to bargaining games of this kind, and show that the solution is uniquely characterised by a set of logical properties. Moreover, we prove that the solution also satisfies the most fundamental game theoretic properties such as symmetry and Pareto optimality. In addition, by lots of simulation experiments we study how the number of conflicting demands, bargainers' risk attitude, and bargainer number influence the bargaining success-rate and efficiency as well as the agreement quality.

Keywords: Bargaining, negotiation, logic, game theory, agent

1. Introduction

Bargaining is a process to settle disputes and reach mutually agreements. It has been investigated from many perspectives, including economics, social science, political science and computer science [20, 4, 25, 23, 11, 8, 30]. Although quantitative approaches dominate bargaining analysis, recently some studies of bargaining in computer science, especially in artificial intelligence, start to pay some attention to logical reasoning behind bargaining processes. In fact, a number of logical frameworks were proposed for specifying reasoning procedures of bargaining [30, 15, 22, 17, 33, 29]. In particular, similar to

*Corresponding author

**Corresponding author

Email addresses: `D.Zhang@uws.edu.au` (Dongmo Zhang), `luoxd3@mail.sysu.edu.cn` (Xudong Luo)

Nash's axiomatic, quantitative model of bargaining, Zhang in [30] proposed an axiomatic model of bargaining in propositional logic. With his model, bargainers' demands are represented in propositional formulae and the outcome of bargaining is viewed as a mutual acceptance of the demands after necessary concessions from each bargainer.

Although Zhang's model provides a purely qualitative approach for bargaining analysis, there is a difficulty to apply his approach to the real-life bargaining. As mentioned in [30], the demands of a player are not necessarily the player's real demands but "*may be the player's beliefs, goals, desired constraints or commonsense*". This could be a problem. For example, a couple bargains over where to go for dinner: either a French restaurant (denoted by f) or an Italian restaurant (denoted by i). The husband prefers Italian food to French food but his wife likes the romantic environment in French restaurants more than Italian ones, even though they have some favourite dishes in common, which may or may not be offered in both restaurants. Obviously each bargainer can express his/her demands in propositional language by writing down their favourite restaurant and dishes, say $\{Italian, pizza\}$. However, if we use Zhang's model, all the domain constraints, such as $(\neg French \vee \neg Italian) \wedge pizza \rightarrow Italian$, have to be included in the demand set of each player, which does not seem intuitive. This paper is devoted to providing a solution to this issue.

Similar to belief merging [13], specifying domain constraints, also known integrity constraints, in a bargaining model gives a number of challenges to the modelling of bargaining reasoning. Firstly, simply assuming logical consistency of individual demand sets is not enough because new constraints may be generated after combining all constraints from individual bargainers as logical consequences. Secondly, preference ordering relies on constraints and therefore a logical requirement for the rationality of preference ordering has to be applied. Finally, constraints and demands from individuals may be described in different forms. It is crucial that a bargaining solution does not rely on the syntax of description, which is actually the case for Zhang's system. As we will see, our model of bargaining is syntax-irrelevant, which in fact reshapes the whole axiomatic system.

The rest of the paper is organised as follows. Section 2 defines our bargaining model. Section 3 introduces our solution concept. Section 4 reveals some properties of our model. Section 5 studies the influence factors for the solution with experiment analysis. Section 6 discusses the related work. Finally, Section 7 summarises our main contributions and points out some possible future work.

2. Bargaining Model

This section presents our bargaining model. We consider a propositional language $\mathcal{L}$ built from a finite set $\mathcal{P}$ of propositional letters and the standard propositional connectives $\{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$. Propositional sentences are denoted by $\phi, \psi, \cdots$. We use $\vdash$ to denote the logical deduction relation in classical propositional logic, and Cn to denote the corresponding local consequence closure. A set K of sentences in $\mathcal{L}$ is *logically closed* if and only if $K = Cn(K)$, where $Cn(K) = \{\phi \mid \phi \in \mathcal{L}, K \vdash \phi\}$.

Further, let Φ be a finite set of propositional formulae in $\mathcal{L}$. We say that Φ is *consistent* if there is no formula ϕ such that $\Phi \vdash \phi$ and $\Phi \vdash \neg\phi$. A binary relation $\geq$ over Φ is a *pre-order* if and only if it is a reflexive and transitive relation over Φ. A pre-order is *total* if for all $\phi, \psi \in \Phi$, $\phi \geq \psi$ or $\psi \geq \phi$. Given a pre-order $\geq$ over Φ, we define $\phi > \psi$ as $\phi \geq \psi$ and $\psi \not\geq \phi$, and $\phi \simeq \psi$ as $\phi \geq \psi$ and $\psi \geq \phi$. Moreover, if $\phi \geq \psi$ then $\psi \leq \phi$; and if $\phi > \psi$ then $\psi < \phi$.

2.1. Bargaining Games

Following [30], we assume that each bargainer has a set of demands and a preference order over the demand set. As we will show later, the domain constraints, commonsense knowledge, and other integrity constraints will be specified separately and so need not to be included in the individual demand set.

Definition 1. *A* **demand structure** *D is a pair $(X, \geq)$, where X is a finite, logically consistent set of demands that are represented by a set of sentences in $\mathcal{L}$, and $\geq$ is a total pre-order on X, which satisfies:*

(LC) *If $\phi_1, \cdots, \phi_n \vdash \psi$, then there exists at least one $k \in \{1, \cdots, n\}$ such that $\psi \geq \phi_k$.*

Intuitively, a demand structure represents the statements a bargainer wants to put into the agreement, and the total pre-order over the demands is the description for bargainer's preference over the demands, i.e., $\phi \geq \psi$ means the bargainer holds demand ϕ more firmly than demand ψ. In addition, the logical constraint LC, introduced by Zhang and Foo in [31], places a rationality requirement on the preference ordering, which says that if a demand of ψ is a logic consequence of demands $\phi_1, \cdots, \phi_n$ then the firmness to keep ψ should not be less than at least one formula in $\phi_1, \cdots, \phi_n$.

The firmness of the demand is analogous to the epistemic entrenchment of the belief in [1]. Consider demands $\phi_1, \cdots, \phi_n$ and ψ such that $\phi_1, \cdots, \phi_n \vdash \psi$. On the one hand, if a bargainer decides to give up demand ψ then he also has to give up at least one ϕ_k (otherwise logical closure will brings ψ back). On the other hand, it is possible to give up $\phi_1, \cdots, \phi_n$ while retain ψ. Hence, ψ is as firm as at least one $\phi_k (k = 1, \cdots, n)$. In this paper, we consider that the demand set of each bargainer is non-empty because no bargainers would participate in the bargaining for nothing.

In a bargaining scenario, an integrity constraint means a rule that all participants in the bargaining must follow. Such a rule could be something like domain restrictions, generic settings, commonsense knowledge and so on. As we will see below, we assume that any integrity constraint can be represented by a propositional formula and all integrity constraints for each bargaining situation are logically consistent. The following definition extends [30]'s bargaining model to allow integrity constraints.

Definition 2. *A* **bargaining game** *is a tuple of $\langle (X_i, \geq_i)_{i \in N}, IC \rangle$, where*

(i) $N = \{1, 2, \cdots, n\}$ is a set of bargainers;

(ii) $(X_i, \geq_i)$ is the demand structure of bargainer i; and

(iii) IC is a consistent set of sentences (i.e., integrity constraints).

The set of all bargaining games in language $\mathcal{L}$ is denoted by $G^{IC}_{n,\mathcal{L}}$.

A bargaining game specifies a snapshot of a bargaining procedure. As we will demonstrate, we model a bargaining procedure as a sequence of bargaining games. Normally, a bargaining starts with a situation in which the demands of the bargainers conflict each other. With the proceeding of negotiation, bargainers may make concessions in order to reach an agreement. Eventually, the bargaining terminates with either an agreement or a disagreement. The terminal situations can be specified in the follow two specific games:

Definition 3. *A bargaining game $\langle (X_i, \geq_i)_{i \in N}, IC \rangle$ is* **non-conflictive** *if $\bigcup_{i=1}^{n} X_i \cup IC$ is logically consistent. It is a* **disagreement** *if there is $k \in N$ such that $X_k = \emptyset$.*

Note that a disagreement means that there is a bargainer who has nothing to give up.[1]

2.2. Demand Hierarchy and Comprehensiveness

Before introducing a solution concept for our bargaining model, we need to introduce a number of concepts about single bargainer's demand structure first.

Definition 4. *Given a demand structure $D = (X, \geq)$ where $X \neq \emptyset$, $P = (X^1, \cdots, X^L)$ is the* **partition** *of D if it satisfies:*

(i) $\forall l \in \{1, \cdots, L\}$, $X^l \neq \emptyset$;

(ii) $X = \bigcup_{l=1}^{L} X^l$;

(iii) $X^k \cap X^l = \emptyset$, *where* $k, l \in \{1, \cdots, L\}$ *but* $k \neq l$; *and*

(iv) $\forall \phi \in X^k$, $\forall \psi \in X^l$, $\phi > \psi$ *if and only if* $k < l$, *where* $k, l \in \{1, \cdots, L\}$.

Now we define the demand hierarchy under IC using the partition above.

Definition 5. *Given a demand structure $D = (X, \geq)$ and a set of integrity constraints IC, let $P = (X^1, \cdots, X^L)$ be a partition of D. Then the* **hierarchy** *of D under IC is defined as follows:*

(i) $H^1 = Cn(X^1 \cup IC)$, *and*

(ii) $H^{k+1} = Cn(\bigcup_{i=1}^{k+1} X^i \cup IC) \setminus \bigcup_{i=1}^{k} H^i$.

$\forall \phi \in Cn(X \cup IC)$, we define $h(\phi) = k$ if and only if $\phi \in H^k$, where k is ϕ's hierarchy level in D. And we write $h_D = \max\{h(\phi) \mid \phi \in Cn(X \cup IC)\}$ as the height of D. In addition, $\forall \phi, \psi \in X$, suppose $\phi \in H^k$ and $\psi \in H^j$, we write

$$\phi \geq^{IC} \psi \quad \textit{iff} \quad k \leq j.$$

For simplicity, we assume that $H^i \neq \emptyset$ for all i. In fact, if there is $k \in N^+$ such that H^k is $\emptyset$, we can remove all empty levels and let the remaining hierarchy as the hierarchy of D. Since $\geq$ is a total pre-order on X, it is easy to see that $\geq^{IC}$ is also a total pre-order on $Cn(X \cup IC)$.

[1] In the real-life bargaining, a bargainer may declare a disagreement when he finds that an agreement would not be reached without giving up all his reservation demands.

Definition 6. *Given a demand structure $D = (X, \geq)$ and a set of integrity constraints IC, Ω is an* **IC-comprehensive set** *of D if:*

(i) $\Omega \subseteq Cn(X \cup IC)$;

(ii) $\Omega = Cn(\Omega)$; *and*

(iii) $\forall \phi \in \Omega$, $\forall \psi \in Cn(X \cup IC)$, $\psi \geq^{IC} \phi$ *implies* $\psi \in \Omega$.

In other words, a subset of $Cn(X \cup IC)$ is IC-comprehensive if it is logically and ordinally closed under $\geq^{IC}$. We denote the set of all IC-comprehensive sets of D by $\Gamma^{IC}(D)$, or $\Gamma(D)$ if IC is obvious from the context.

The following theorem is important to our bargaining solution.

Theorem 1. *Given a demand structure $D = (X, \geq)$ and a set of integrity constraints IC, a set Ω is an IC-comprehensive set of D if and only if there exists $k \in \{1, \cdots, h_D\}$ such that $\Omega = \bigcup_{i=1}^{k} H^i$.*

Proof. ($\Rightarrow$) We first prove that if $\Omega \in \Gamma(D)$ then there exists $k \in \{1, \cdots, h_D\}$ such that $\Omega = T^k$, where $T^k = \bigcup_{i=1}^{k} H^i$. Let $k^0 = \min\{k \mid \Omega \subseteq T^k\}$. Obviously, $1 \leq k^0 \leq h_D$. We aim to prove $\Omega = T^{k^0}$. By the definition of k^0, $\Omega \subseteq T^{k^0}$. So, we just need to prove $T^{k^0} \subseteq \Omega$. Suppose it is not the case. Then there must exist ψ such that $\psi \in T^{k^0}$ but $\psi \notin \Omega$. Since $\Omega \in \Gamma(D)$, $\forall \phi \in \Omega$, we have $\psi \in Cn(X \cup IC)$ and $\psi \notin \Omega$, and then $\psi <^{IC} \phi$. So, $h(\psi) > h(\phi)$. $\forall \phi \in \Omega$, we have $1 \leq h(\phi) \leq k^0$. In addition, $k^0 = \min\{k \mid \Omega \subseteq T^k\}$. Therefore, $h(\psi) > k^0$. However, $\psi \in T^{k^0}$, and then $1 \leq h(\psi) \leq k^0$, which is contradicting. Therefore, the assumption is false, i.e., we have $T^{k^0} \subseteq \Omega$. As a result, if $\Omega \in \Gamma(D)$, we can find $k = k^0$ such that $\Omega = T^{k^0}$.

($\Leftarrow$) $\forall k \in \{1, \cdots, h_D\}$, we need to prove $T^k \in \Gamma(D)$. Because

$$T^k = \bigcup_{i=1}^{k} H^i = Cn(\bigcup_{i=1}^{k} X^i \cup IC),$$

we know that T^k is closed. In addition, $\forall k \in \{1, \cdots, h_D\}$, $\bigcup_{i=1}^{k} X^i \subseteq X$, and then

$$T^k = Cn(\bigcup_{i=1}^{k} X^i \cup IC) \subseteq Cn(X \cup IC).$$

$\forall k \in \{1, \cdots, h_D\}$, $\forall \phi \in T^k$, and $\psi \in Cn(X \cup IC)$, if $\psi \geq^{IC} \phi$, we need to prove $\psi \in T^k$. Suppose it is not this case, i.e., $\psi \notin T^k$. Because

$$T^k = Cn(X \cup IC) \backslash \bigcup_{i=k+1}^{h_D} H^i,$$

then $\psi \in \bigcup_{i=k+1}^{h_D} H^i$, and so $k+1 \leq h(\psi) \leq h_D$. In addition, since $\phi \in T^k$, $T^k = \bigcup_{i=1}^{k} H^i$, $1 \leq h(\phi) \leq k$. So, we can get $h(\psi) > h(\phi)$, and then $\phi >^{IC} \psi$, which contradicts premise $\psi \geq^{IC} \phi$. Therefore, the assumption is false. □

The following defines the equivalence of demand structures under integrity constraints, which plays an important role for describing syntax independency:

Definition 7. *Let $D = (X, \geq)$ and $D' = (X', \geq')$ be two demand structures, where $X \neq \emptyset$ and $X' \neq \emptyset$, IC is a set of integrity constraints. We say D and D' are* **equivalent** *under IC, denoted as $D \Leftrightarrow^{IC} D'$, if and only if there is $\Gamma(D) = \Gamma(D')$.*

3. Bargaining Solution

In this section, we will develop our solution concept for the bargaining model we introduced in the previous section.

Definition 8. *A* **solution** *to bargaining game G, denoted as S, is a function from $G^{IC}_{n,\mathcal{L}}$ to $\prod_{i=1}^{n} \Gamma(D_i)$, i.e.,*

$$\forall G \in G^{IC}_{n,\mathcal{L}}, S(G) = (s_1(G), \cdots, s_n(G)), \tag{1}$$

where $s_i(G) \in \Gamma(D_i)$ for each i. Then the agreement of the bargaining game is defined as:

$$A(G) = Cn(\bigcup_{i=1}^{n} s_i(G)). \tag{2}$$

Intuitively, the agreement of a bargaining is a set of demands mutually accepted by all the bargainers. A bargaining solution is then to specify which demands from each bargainer should be put into the final agreement.

In the following, we will construct a concrete bargaining solution that satisfies a set of desirable properties. The intuition behind the construction can be stated as follows: assume a bargaining situation where all bargainers agree on a set of integrity constraints IC. Firstly, all the bargainers submit their demands to an arbitrator who also knows IC. The arbitrator then judges if the current bargaining situation forms a non-conflictive game or a disagreement game. If so, the bargaining stops with either an agreement, which is the collection of all the demands, or a disagreement, which is an empty set. Otherwise, the arbitrator requests each bargainer to make a concession by withdrawing their least preferred demands. We call such a solution a simultaneous concession solution. Formally, we have:

Definition 9. *Given a bargaining game $G = \langle (X_i, \leq_i)_{i\in N}, IC\rangle$, its* **simultaneous concession solution***, denoted as $S_{sc}(G)$, is constructed as follows:*

$$S_{sc}(G) = \begin{cases} (H_1^{\leq h_{D_1}-\rho}, \cdots, H_n^{\leq h_{D_n}-\rho}) & \text{if } \rho < L, \\ (\emptyset, \cdots, \emptyset) & \text{otherwise,} \end{cases} \tag{3}$$

where $\forall i \in N$, $H_i^{\leq j} = \bigcup_{k=1}^{j} H_i^k$ (H^k is defined in Definition 5), h_{D_i} is the height of D_i, $\rho = \min\{k \mid \bigcup_{i=1}^{n} H_i^{\leq h_{D_i}-k}$ is consistent$\}$, and $L = \min\{h_{D_i} \mid i \in N\}$.

For a better understanding of the above definition, let us discuss the restaurant example in the introduction again.

Example 1. *A couple bargains over which restaurant to go to celebrate their wedding anniversary: either Italian (i) or French (f). The husband (h) likes to eat pizza (p). Alternatively, he is also fine with beefsteak (b) and vegetable salads (v). In fact, he does not mind to go to the French restaurant but cannot bear people eating snails (s). The wife (w) leans towards the romantic French restaurant and particular likes the vegetable salads. She would like to try snails once as all her friends recommend it. Both know that pizza is only offered in the Italian restaurant ($p \to i$) and snails only offered in the French restaurant ($s \to f$). Obviously they can only choose one restaurant for the dinner ($\neg i \vee \neg f$).*

Putting all the information together, the husband's demands can be written as:

$$X_h = \{\neg s, p, v, b\}$$

with the preference:

$$\neg s \geq_h p \geq_h v \geq_h b;$$

Husband	**Wife**
$\neg s$, $\neg i \vee \neg f$, $p \to i$, $s \to f$	
p, i, $\neg f$	v, $\neg i \vee \neg f$, $p \to i$, $s \to f$
v	f, $\neg i$, $\neg p$
b	s

Table 1: Player's hierarchies from high (top) to low (bottom).

the wife's demands are

$$X_w = \{v, f, s\}$$

with the preference:

$$v \geq_w f \geq_w s;$$

and the integrity constraints can be represented by

$$IC = \{\neg i \vee \neg f, p \to i, s \to f\}.$$

Thus, we can model the game as:

$$G = \langle (X_h, \geq_h), (X_w, \geq_w), IC \rangle.$$

To solve the problem, we first calculate the normalised hierarchy for each player according to Definition 5 as shown in Table 1. From the table, it is easy to see that $h_{D_h} = 4$, $h(D_w) = 3$, $\rho = 2$, and $L = 3$. Then the solution of the bargaining game is:

$$\begin{aligned} s_h(G) &= H_h^{\leq 2} = \{\neg s, \neg i \vee \neg f, p \to i, s \to f, p, i, \neg f\}, \\ s_w(G) &= H_w^{\leq 1} = \{v, \neg i \vee \neg f, p \to i, s \to f\}. \end{aligned}$$

As a result, the agreement of the bargaining is:

$$A(G) = Cn(H_h^{\leq 2} \cup H_w^{\leq 1}) = \{\neg s, \neg i \vee \neg f, p \to i, s \to f, p, i, \neg f, v\}.$$

4. Properties of the Solution

In this section, we investigate the properties of the solution that we construct in the previous section. To this end, we need to introduce a few concepts first.

Definition 10. *Given two bargaining games* $G = \langle (D_i)_{i\in N},\ IC\rangle$ *and* $G' = \langle (D'_i)_{i\in N},\ IC'\rangle$*, we say* G *and* G' *are* **equivalent***, denoted by* $G \equiv G'$*, if and only if*

(i) both G *and* G' *are disagreement games; or*

(ii) none of G *and* G' *is a disagreement game,* $\vdash IC \leftrightarrow IC'$ *and* $D_i \Leftrightarrow^{IC} D'_i$ $\forall i \in N$*.*

Definition 11. *Given a bargaining game* $G = \langle (D_i)_{i\in N}, IC\rangle$*, a bargaining game* $G' = \langle (D'_i)_{i\in N}, IC'\rangle$*, where* D_i=$(X'_i, \geq_i)$*, is a* **subgame** *of* G*, denoted by* $G' \subseteq G$*, if and only if* $\forall i \in N$*,*

(i) $IC \vdash IC'$ *and* $IC' \vdash IC$ *(we write it simply as* $\vdash IC \leftrightarrow IC'$*);*

(ii) $Cn(X'_i \cup IC')$ *is an IC-comprehensive set of* D_i*; and*

(iii) $\geq'^{IC'}_i = \geq^{IC}_i \cap (Cn(X'_i \cup IC') \times Cn(X'_i \cup IC'))$*.*

Furthermore, G' *is a proper subgame of* G*, denoted by* $G' \subset G$*, if* $Cn(X'_i \cup IC') \subset Cn(X_i \cup IC)$ $\forall i \in N$*.*

Intuitively, each subgame represents a stage in G or these games that are equivalent to that stage. However, it does not mean that any game has a proper subgame. For example, given a bargaining game $G = \langle (D_i)_{i\in N}, IC\rangle$, and $h_{D_i} = 1$ for any D_i in G, then G does not have any proper subgame. Of course, a game could have more than one proper subgames. Thus, we need the following concept:

Definition 12. *A proper subgame* G' *of* G *is a* **maximal proper subgame** *of* G*, denoted by* $G' \subset_{max} G$*, if for any* $G'' \subset G$*,* $G'' \subseteq G'$*.*

4.1. Logical Characterisation

We first consider the logical properties of our bargaining solution. In general, we expect any bargaining solution to satisfy some intuitions as follows. (i) If the integrity constraints are consistent, then the outcome of the bargaining, i.e., the agreement, should also be consistent. (ii) If there is no conflict among all the bargainers' demands and the integrity constraints, then nobody has to make any concession to reach an agreement. (iii) A disagreement means that no agreement is reached. (iv) If two bargaining games are equivalent, then the solutions of bargaining are the same. This property is crucial for the bargaining, while as we can see, it is not satisfied in [30]. (v) A bargaining solution should be independent of any minimal simultaneous concession of the bargaining game. In this subsection, we will show that our simultaneous concession solution satisfies all the five properties.

Firstly we need the following lemma:

Lemma 1. *Given two bargaining games G (where $h_{D_i} > 1$ for any D_i in G) and G', G' is a maximal proper subgame of G if and only if $\forall i$,*

(i) $\vdash IC \leftrightarrow IC'$;

(ii) $Cn({X'}_i \cup IC') = H_i^{\leq h_{D_i}-1}$; *and*

(iii) ${\geq'}_i^{IC'} = \geq_i^{IC} \cap (Cn({X'}_i \cup IC') \times Cn({X'}_i \cup IC'))$.

Proof. ($\Leftarrow$) We will prove that if G' satisfies properties (i)-(iii). G' is a maximal proper subgame of G. Because property (ii) is satisfied, G is not a disagreement. Thus, we need to prove $G' \subset G$ first. Since we find properties (i) and (iii) are the same as (i) and (iii) in Definition 11, we just need to prove (ii), i.e., $Cn({X'}_i \cup IC') = H_i^{\leq h_{D_i}-1}$ is an IC comprehensive set of D_i and $Cn({X'}_i \cup IC') \subset Cn(X_i \cup IC)$. Because $h_{D_i} > 1$ for any D_i in G, by Theorem 1, $H_i^{\leq h_{D_i}-1} = \bigcup_{j=1}^{h_{D_i}-1} H_i^j$ is an IC comprehensive set of D_i. In addition, since $H_i^{h_{D_i}} \neq \emptyset$, $T_i^{h_{D_i}-1} = Cn(X_i \cup IC) \backslash H_i^{h_{D_i}} \subset Cn(X_i \cup IC)$.

Next, for $G'' = \langle {D''}_{i\in N}, \geq'' \rangle$, if $G'' \subset G$, we need to prove $G'' \subseteq G'$. Because $G'' \subset G$ and $G' \subset G$, (a) $IC'' \leftrightarrow IC \leftrightarrow IC'$; (b) $Cn(X''_i \cup IC'')$ and $Cn(X'_i \cup IC')$ are IC comprehensive sets of D_i for all i; and (c) ${\geq''}_i^{IC} = \geq_i^{IC}$ $\cap (Cn({X''}_i \cup IC'') \times Cn({X''}_i \cup IC'')) = {\geq'}_i^{IC} \cap (Cn(X''_i \cup IC'') \times Cn(X''_i \cup IC''))$. In addition, $Cn(X''_i \cup IC'') \subset Cn(X_i \cup IC)$ and $Cn(X'_i \cup IC'') \subset Cn(X_i \cup IC)$ for all i. So, we just need to prove $Cn(X''_i \cup IC'')$ is an IC comprehensive set of D'_i for all i.

We prove $Cn(X''_i \cup IC'') \subseteq Cn(X'_i \cup IC')$ first. Suppose not. Then there is $\phi \in Cn(X''_i \cup IC'')$ but $\phi \notin Cn(X'_i \cup IC')$. From $Cn(X'_i \cup IC') = H_i^{\leq h_{D_i}-1} = Cn(X_i \cup IC) \backslash H_i^{h_{D_i}}$ and $Cn(X''_i \cup IC'') \subset Cn(X_i \cup IC)$, we derive that $\phi \in H_i^{h_{D_i}}$. Therefore, $h(\phi) = h_{D_i}$. Because $\forall \psi \in Cn(X_i \cup IC)$, $h(\psi) \leq h_{D_i}$, $\psi \geq^{IC} \phi$, and $Cn(X''_i \cup IC'')$ is an IC-comprehensive set of D_i, $\psi \in Cn(X''_i \cup IC'')$, which implies $Cn(X_i \cup IC) \subseteq Cn(X''_i \cup IC'')$. However, this contradicts $Cn(X_i'' \cup IC'') \subset Cn(X_i \cup IC)$. So, the assumption cannot hold. Thus, $Cn(X_i'' \cup IC'') \subseteq Cn(X_i' \cup IC')$.

Because $G' \subset G$ and $G'' \subset G$, for all i, $Cn(X_i' \cup IC')$ and $Cn(X_i'' \cup IC'')$ are both IC-comprehensive sets of D_i. Thus, $\forall \phi \in Cn(X_i'' \cup IC'')$, $\forall \psi \in Cn(X_i' \cup IC')$ (thus we have $\psi \in Cn(X_i \cup IC)$), if $\psi \geq^{IC} \phi$ then $\psi \in Cn(X_i'' \cup IC'')$. Therefore, $Cn(X_i'' \cup IC'')$ is an IC comprehensive set of D_i'. Furthermore, $G'' \subseteq G'$.

($\Rightarrow$) If G' is a maximal proper subgame of G, then properties (i)–(iii) in this lemma are satisfied.

Firstly, because G has at least one proper subgame G', $\forall i \in N$, $X_i \neq \emptyset$ and $Cn(X_i' \cup IC') \subset Cn(X_i \cup IC)$. If G' is a maximal proper subgame of G, by Definitions 11 and 12, properties (i) and (iii) of this lemma are satisfied, and so we just need to prove its property (ii). Suppose not, i.e., $Cn(X'_i \cup IC') \neq H_i^{\leq h_{D_i}-1}$. Because G' is a proper subgame of G, $Cn(X_i' \cup IC')$ is an IC comprehensive set of D_i. Thus, by Theorem 1, there exists $k \in [1, h_{D_i}]$, such that $Cn(X'_i \cup IC') = \bigcup_{j=1}^{k} H_i^j$. So, because $Cn(X_i' \cup IC') \subset Cn(X_i \cup IC)$ and $Cn(X'_i \cup IC') \neq H_i^{\leq h_{D_i}-1}$, we have $k < h_{D_i} - 1$.

Here we can find a subgame of G'' as proved above such that: (a) $\vdash IC \leftrightarrow IC''$; (b) $Cn(X''_i \cup IC'') = H_i^{\leq h_{D_i}-1}$; and (c) $\geq''^{IC''}_i = \geq_i^{IC} \cap (Cn(X''_i \cup IC'') \times Cn(X''_i \cup IC''))$. Then $G'' \subset G$. Thus, because

$$k < h_{D_i} - 1, \bigcup_{j=1}^{k} X_i^j \subset \bigcup_{j=1}^{h_{D_i}-1} X_i^j,$$

we have:

$$\bigcup_{j=1}^{k} H_i^j = Cn(\bigcup_{j=1}^{k} X_i^j \cup IC) \subset Cn(\bigcup_{i=1}^{h_{D_i}-1} X_i^j \cup IC) = H_i^{\leq h_{D_i}-1}.$$

So, we have $G'' \supset G'$. This contradicts that G' is a maximal proper subgame of G. So, the assumption is false. Thus, $Cn(X'_i \cup IC') = H_i^{\leq h_{D_i}-1}$. □

Now we are ready to prove our solution satisfies the five properties, which reflect basic requirements that a solution should meet:

Theorem 2. *Given a bargaining game $G = \langle (X_i, \leq_i)_{i \in N}, IC \rangle$, let its simultaneous concession solution be $S_{sc}(G)$ and its agreement be $A(G)$. Then the following properties hold:*

(i) Consistency: If IC is consistent, then $A(G)$ is consistent.

(ii) Non-conflictive: If G is non-conflictive, then $S_{sc}(G) = (Cn(X_i \cup IC)_{i\in N}$.

(iii) Disagreement: If G is a disagreement, then $A(G) = \emptyset$.

(iv) Equivalence: If $G \equiv G'$, then $S_{sc}(G) = S_{sc}(G')$.

(v) Contraction independence: If $G' \subset_{max} G$ then $S_{sc}(G) = S_{sc}(G')$ unless G is non-conflictive.

Proof. (i) Suppose IC is consistent. Given an IC bargaining game G, if $\rho \geq L$ then $S_{sc}(G) = (\emptyset, \cdots, \emptyset)$. So, $A(G) = \emptyset$. Obviously, it is consistent. If $\rho < L$, then

$$S_{sc}(G) = (H_1^{\leq h_{D_1}-\rho}, \cdots, H_n^{\leq h_{D_n}-\rho}),$$

because by Definition 9, we have

$$\rho = \min\{k \mid \bigcup_{i=1}^{n} H_i^{\leq h_{D_i}-k} \text{ is consistent}\}.$$

So, $A(G) = Cn(\bigcup_{i=1}^{n} H_i^{\leq h_{D_i}-\rho})$ is consistent. That is, the consistency property holds.

(ii) If G is non-conflictive, by Definition 3, $\bigcup_{i=1}^{n} X_i \cup IC$ is consistent. Then we can easily get $\rho = 0$ and $L \geq 1$. Thus, we have:

$$S_{sc}(G) = (H_i^{\leq h_{D_i}-0})_{i\in N} = (H_i^{\leq h_{D_i}})_{i\in N}.$$

Then, noticing $h_{D_i} = \max\{h(\phi) \mid \forall\phi \in Cn(X_i \cup IC)\}$, we have:

$$H_i^{\leq h_{D_i}} = Cn(\bigcup_{j=1}^{h_{D_i}} X_i^j \cup IC) = Cn(X_i \cup IC).$$

So, $S_{sc}(G) = (Cn(X_i \cup IC))_{i\in N}$.

(iii) If G is a disagreement, by Definition 3, there exists k such that $X_k = \emptyset$, and then $L = 0$; but $\rho \geq 0$. So, $\rho \geq L$. Thus, $s_i(G) = \emptyset$ for any i. Furthermore, $A(G) = Cn(\bigcup_{i=1}^{n} s_i(G)) = \emptyset$.

(iv) Given two bargaining games $G = \langle (X_i, \geq_i)_{i\in N}, IC\rangle$ and $G' = \langle (X_i', \geq_i')_{i\in N}, IC'\rangle$ such that $G \equiv G'$. By Definitions 10 and 7, if G is a disagreement, so is G'; otherwise, $IC \leftrightarrow IC$ and $\Gamma(D) = \Gamma(D')$. Therefore, if G is non-conflictive, it is easy to see that G' is non-conflictive. So, $S_{sc}(G) = S_{sc}(G')$. Otherwise, we can easily have $h_{D_i} = h_{D'_i}$, $\rho = \rho'$ and $L = L'$. In addition, since $H_i^{\leq h_{D_i} - \rho} = {H'}_i^{\leq h_{D'_i} - \rho'}$, we have $S_{sc}(G) = S_{sc}(G')$. That is, the equivalence property holds.

(v) Consider a bargaining game $G = \langle (X_i, \geq_i)_{i \in N}, IC \rangle$. (a) If $L = 0$, then $\exists k, X_i^k = \emptyset$, which means G has no proper subgames. Then $S_{sc}(G)$ satisfies the contraction independence property trivially. (b) If $L > 0$ then because G is not non-conflictive, $\rho > 0$. Assume $G' = \langle (X'_i, \geq'_i)_{i \in N}, IC' \rangle$ is a maximal proper subgame of G. Let $\rho' = \min\{k' \mid \bigcup_{i=1}^{n} {H'}_i^{\leq h_{D'_i} - k'}$ is consistent$\}$ and $L' = \min\{h'_{D_i} \mid i \in N\}$. By Lemma 1, $\forall i \in N, \vdash IC \leftrightarrow IC'$; $Cn({X'}_i \cup IC') = H_i^{\leq h_{D_i} - 1}$; and ${\geq'}_i^{IC'} = \geq_i^{IC} \cap (Cn({X'}_i \cup IC') \times Cn({X'}_i \cup IC'))$. Obviously, $L' = L - 1$ and $\rho' = \rho - 1$, and $h_{D'_i} = h_{D_i} - 1$. Therefore, $\rho' < L'$ if and only if $\rho < L$. If $\rho \geq L$, $S_{sc}(G) = S_{sc}(G') = (\emptyset, \cdots, \emptyset)$. Otherwise, when $\rho < L$, $\forall i \in N$, we have

$$
\begin{aligned}
S_{sc}(G) &= \left(H_i^{\leq h_{D_i} - \rho}\right)_{i \in N} \\
&= \left(Cn(X_i \cup IC) \backslash \cup_{j = h_{D_i} - \rho + 1}^{h_{D_i}} H_i^j\right)_{i \in N} \\
&= \left(\left(Cn(X_i \cup IC) \backslash H_i^{h_{D_i}}\right) \backslash \cup_{j = h_{D_i} - \rho + 1}^{h_{D_i} - 1} H_i^j\right)_{i \in N} \\
&= \left(H_i^{\leq h_{D_i} - 1} \backslash \cup_{j = h_{D'_i} - \rho' + 1}^{h_{D'_i}} H_i^j\right)_{i \in N} \\
&= \left(Cn({X'}_i \cup IC') \backslash \cup_{j = h_{D'_i} - \rho' + 1}^{h_{D'_i}} H_i^j\right)_{i \in N} \\
&= \left(H_i^{\leq h_{D'_i} - \rho'}\right)_{i \in N} \\
&= S_{sc}(G').
\end{aligned} \tag{4}
$$

Therefore, the contraction independence property holds. □

The following theorem shows that the five properties exactly characterise the simultaneous concession solution (therefore putting these two theorems together forms a representation theorem of our solution).

Theorem 3. *If a bargaining solution s satisfies the properties of consistency, non-conflictive, disagreement, equivalence, and contraction independence, it is the simultaneous concession solution.*

Proof. For any G by induction on ρ, we prove that if a bargaining solution $S(G)$ satisfies the five properties, it is the simultaneous concession solution $S_{sc}(G)$, i.e., $S(G) = S_{sc}(G)$.

For the base case that $\rho = 0$, there are two situations. (i) If G is non-conflictive, according to the non-conflictive property, then we have:

$$S(G) = (Cn(X_i \cup IC))_{i \in N} .$$

Because $\rho = 0$ and $L \geq 1$, $\rho < L$. Thus, by Definition 9, $\forall i \in N$, we have

$$\begin{aligned} S_{sc}(G) &= \left(H_i^{\leq h_{D_i} - 0}\right)_{i \in N} \\ &= \left(H_i^{\leq h_{D_i}}\right)_{i \in N} \\ &= \left(Cn\left(\bigcup_{j=1}^{h_{D_i}} X_i^j \cup IC\right)\right)_{i \in N} \\ &= (Cn(X_i \cup IC))_{i \in N} . \end{aligned}$$

So, $S(G) = S_{sc}(G)$. (ii) If G is a disagreement, by the disagreement property, we have

$$S(G) = (\emptyset)_{i \in N}$$

and there must exist a k such that $X_k = \emptyset$ and so $L = 0$. Since $\rho = 0$, $\rho = L$. Thus, by Definition 9, $S_{sc}(G) = (\emptyset)_{i \in N}$. So, $S(G) = S_{sc}(G)$.

Now we assume that for any game G' such that $\rho' = k$, $S(G') = S_{sc}(G')$. Now for a game G in which $\rho = k + 1$, we aim to prove $S(G') = S_{sc}(G')$. Because $\rho = k + 1 \geq 1$ in G, G is not a disagreement game nor a non-conflictive game. Let $G' = \langle (X'_i, \geq'_i)_{i \in N}, IC' \rangle$, where: (a) $\vdash IC \leftrightarrow IC'$, (b) $Cn({X'}_i \cup IC') = H_i^{\leq h_{D_i} - 1}$, and (c) $\geq'^{IC'}_i = \geq^{IC}_i \cap (Cn({X'}_i \cup IC') \times Cn({X'}_i \cup IC'))$ for any i. So, G' is a maximal proper game of G. Here $\rho' = \rho - 1 = k$, so by inductive assumption, we have $S(G') = S_{sc}(G') = (H_i^{\leq h_{D'_i} - \rho'})_{i \in N}$. In addition, by the contraction independence property, $S(G') = S(G)$. So, we just need to prove

$$S_{sc}(G) = \left(H_i^{\leq h_{D_i} - \rho}\right)_{i \in N} = \left(H_i^{\leq h_{D'_i} - \rho'}\right)_{i \in N},$$

which is similar to formula (4). $\square$

4.2. Game-Theoretic Properties

In this subsection, we show that our solution satisfies two fundamental game-theoretical properties: Pareto efficiency and symmetry. Because game-theoretical bargaining model is based on utility functions but our bargaining model is defined on bargainers' demands, we need to restate Pareto efficiency and symmetry for our model. Firstly we need two relevant definitions.

Definition 13. *Given bargaining game $G = \langle (D_i)_{i\in N}, IC\rangle$, where $D_i = (X_i, \geq_i)$, an* **outcome** *of G is a tuple of $O = (o_1, \cdots, o_n)$, where $\forall o_i \in \Gamma(D_i)$, $\bigcup_{i=1}^{n} o_i$ is consistent.*

Definition 14. *Given two bargaining games $G = \langle D, IC\rangle$ and $G' = \langle D', IC'\rangle$, where $D = (X_i, \geq_i)_{i\in N}$ and $D' = (X'_i, \geq'_i)_{i\in N}$. We say G and G' are* **symmetric** *if and only if there is a bijection g from D to D' such that $\forall i \in N, g(D_i) \Leftrightarrow^{IC} D_i$, and $\vdash IC \leftrightarrow IC'$.*

Theorem 4. *Let simultaneous concession solution $S_{sc}(G)$ satisfies:*

(i) Weak Pareto efficiency: Given bargaining game $G = \langle (X_i, \geq_i)_{i\in N}, IC\rangle$ satisfying $S_{sc}(G) \neq (\emptyset, \cdots, \emptyset)$, let $O = (o_i)_{i\in N}$ and $O' = (o'_i)_{i\in N}$ be two possible outcomes of G. If $o'_i \supset o_i$ for all $i \in N$, then $S_{sc}(G) \neq O$.

(ii) Symmetry: Suppose that two bargaining games G and G' are symmetric with bijection g. Then $A(G) = A(G')$. Moreover, $\forall i, j \in N$, if $g(D_i) = D'_j$, then $s_i(G) = s_j(G')$.

Proof. Firstly, we prove our simultaneous concession solution S_{sc} satisfies the weak Pareto efficiency by the contradiction proof method. Suppose $S_{sc}(G) = O$, then by Definition 9, $O = (H_1^{\leq h_{D_1}-\rho}, \cdots, H_n^{\leq h_{D_n}-\rho})$, where $\rho = \min\{k \mid \bigcup_{i=1}^{n} H_i^{\leq h_{D_i}-k}$ is consistent$\}$. Because $o'_i \supset o_i$ for all $i \in N$, $o'_i = H_i^{\leq h_{D_i}-\rho'}$, where $h_{D_n} - \rho' > h_{D_n} - \rho$. Then $\rho' < \rho$. By the definition of ρ, $\bigcup_{i=1}^{n} H_i^{\leq h_{D_i}-\rho'}$ is inconsistent, which means that $\bigcup_{i=1}^{n} o'_i$ is inconsistent, and then O' is not an outcome of G. This conclusion is conflict with the premise. So, $S_{sc}(G) \neq O$.

Then we prove the simultaneous concession solution satisfies the symmetry.

(i) If G is a disagreement, there exist at least $X_k (k \in N)$ in D_k such that $X_k = \emptyset$, and so $L = 0$ for bargaining game G. Because G and G' are symmetric, there must be a $D'_{k'}$ in G' such that $D'_k \Leftrightarrow^{IC} D'_{k'}$, $X'_{k'}(k' \in N) = \emptyset$, and so $L' = 0$ for bargaining game G'. Thus, $\rho \geq L$ and $\rho' \geq L'$ in G and G', respectively. Therefore, by Definition 9, $S_{sc}(G) = S_{sc}(G') = \emptyset$.

(ii) In the case that G and G' are not disagreements, since G and G' are symmetric, and $g(D_i) = D'_j$, by Definition 14, $D_i \Leftrightarrow^{IC} D'_j$ for any $i, j \in N$, and $\vdash IC \leftrightarrow IC'$. Then for any D_i in G and D'_j in G', $\forall \Omega \in \Gamma(D_i)$, $\exists \Omega' \in \Gamma(D'_j)$ such that $\Omega = \Omega'$; and vice visa. It is easy to see that $\rho = \rho'$, $h_{D_i} = h_{D'_j}$ and $L = L'$ for G and G', respectively. Then:

$$\forall i, j \in N, s_i(G) = H_i^{\leq h_{D_i} - \rho} = {H'}_j^{\leq h_{D'_i} - \rho'} = s_j(G').$$

In this case, we have

$$A(G) = Cn(\bigcup_{i=1}^{n} s_i(G)) = Cn(\bigcup_{j=1}^{n} s_j(G')) = A(G').$$

□

5. Experimental Analysis

In the previous section, we theoretically study the properties of the bargaining solution. Now in this section, we will carry out an experimental study and further reveal some insights into our model. More specifically, we conduct several experiments to see how our model works and how good our solution concept is. In each experiment, we do 1000 times under the setting according to the bargaining model (see Definition 2) and bargaining solution (see Definition 9).

We consider three factors that could influence the solution of a bargaining: (i) the number of the conflicting demands in a bargaining game, (ii) the risk attitude of the bargainers, and (iii) the number of the bargainers. Furthermore, for each factor, we will evaluate our model against three different criteria:

(i) *Bargaining Success-rate*. It is the percetage of the bargaining games in which an agreement is successfully reached at the end. As we have defined in Definition 3, no agreements can be reached at the end of the bargaining if there is at least one bargainer who has nothing to give up.

(ii) *Bargaining Efficiency.* It reflects how many steps the bargainers need to reach an agreement. So, it can be measured by the averaging number of the rounds in all the games in which the bargainers have to make concessions by giving up their least preferred demands.

(iii) *Agreement quality.* It can be indicated by the averaging number of the demands in the agreements after bargainings. Intuitively, the more demands are remained in the agreement at the end of a bargaining, the more the bargainers will be satisfied with the agreement, i.e., the higher the agreement quality is.

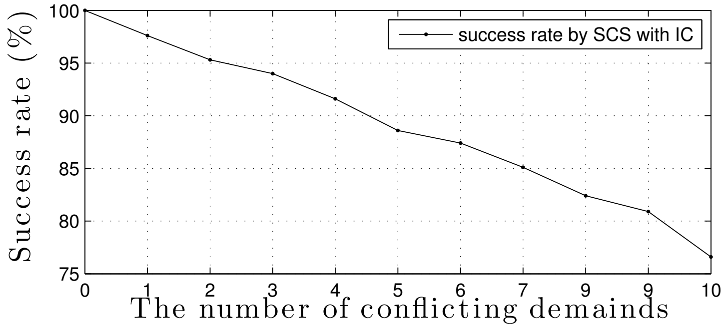
5.1. The Number of Conflicting Demands

According to the bargaining solution in Definition 9, we can see that the conflicting demands play the main role in determining the final agreement among bargainers. Therefore, we should investigate how the result of the bargaining changes with the number of the conflicting demands. More specifically, we conduct the experiments between two bargainers with two integrity constraints, and for each bargainer, we randomly set 14 demands in different preference levels.

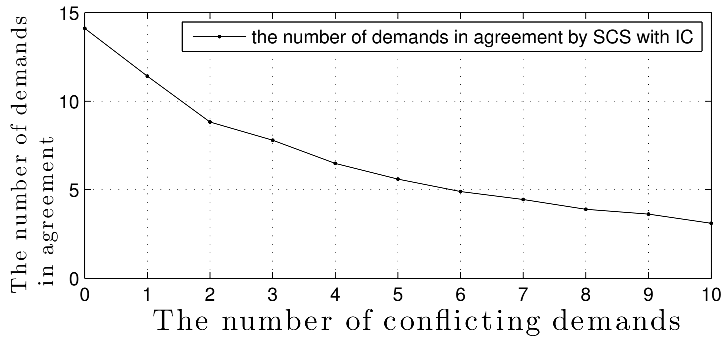
The experimental results are shown in Fig. 1. From Figs. 1(a) and (b), we can see that when the conflicting demands are increasing, the bargaining success-rate and the demands remained in the agreement of the bargaining both decrease. However, the declining rate of the agreement quality is more slowly compared with that of the bargaining success-rate. Thus, we can conclude that the number of the conflicting demands has a greater impact upon the bargaining success-rate than on the agreement quality. In addition, Fig. 1(c) reveals that the more conflicting demands in the bargaining, the more concessions the bargainers have to make in order to reach an agreement.

5.2. Risk Attitude

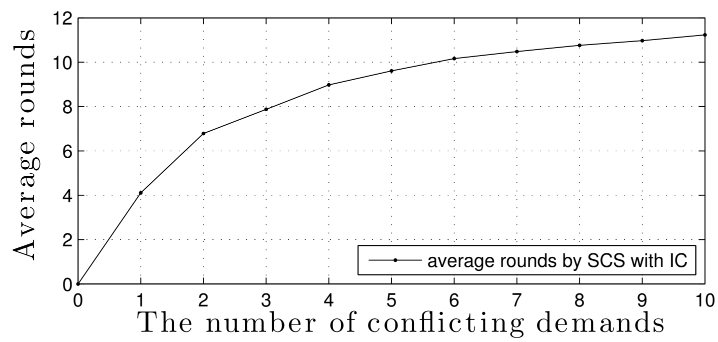
A bargainer's risk attitude reflects his risk posture and determines his bargaining power. In the game-theoretic model of bargaining [20], risk-seeking bargainers have some advantage over risk-averse ones [24]. In fact, like the logical framework of bargaining in [30, 33], in our model the bargainers' attitudes towards risk are specified in the following way: a risk-averse bargainer would give lower priorities to the demands that likely conflict with the demands of the other bargainers and the integrity constraints so that an



(a)



(b)



(c)

Figure 1: The bargaining success-rate and efficiency as well as the agreement quality change with the number of the conflicting demands, respectively.

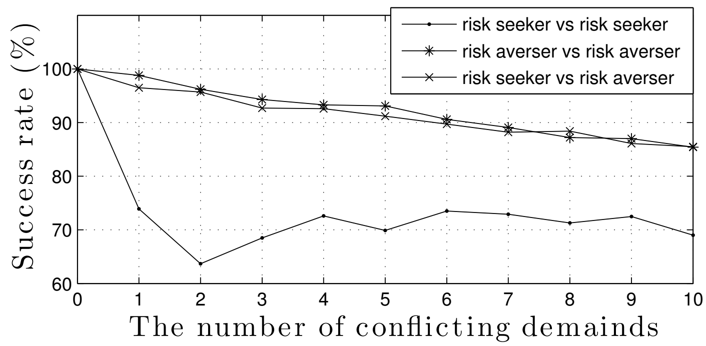


Figure 2: Bargaining success-rate

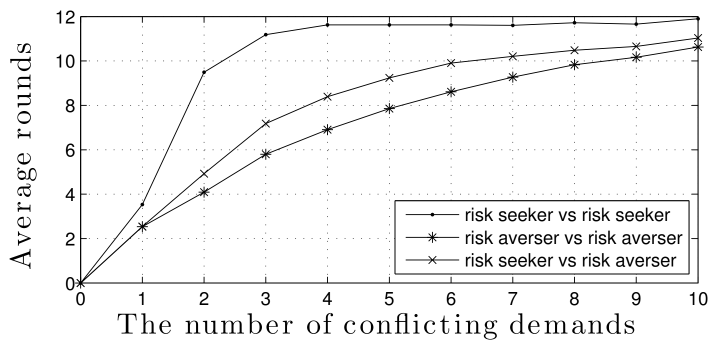


Figure 3: Bargaining efficiency

agreement is more likely to be reached; in contrast, a risk-seeking bargainer would put the conflict demands in higher hierarchies.

This subsection will study how the risk attitudes of the bargainers impact upon the bargaining solutions. To this end, we set up the experiments similar to the above experiments, but consider the two bargainers in the three combination types: (i) both are risk-seeking, (ii) one is risk-seeking and the other is risk-averse, and (iii) both are risk-averse.

From Fig. 2, we can see that no matter how many conflicting demands in the beginning of each bargaining, the success-rate of the bargaining games with both risk-seeking bargainers is much lower than that in the bargaining games with risk-averse bargainers (both are risk-averse or one of the bargainer is risk-averse). This result is consistent with our intuition because the risk-averse bargainers will choose to give up the demands that more likely

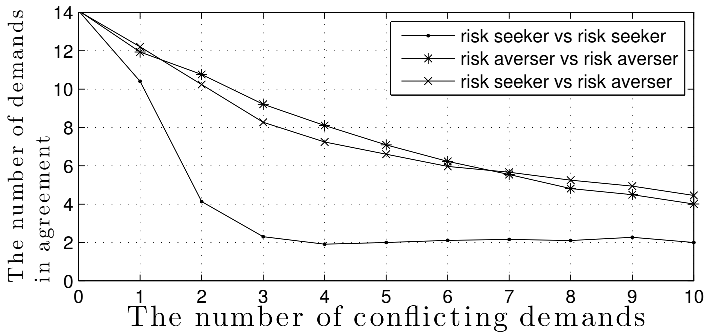
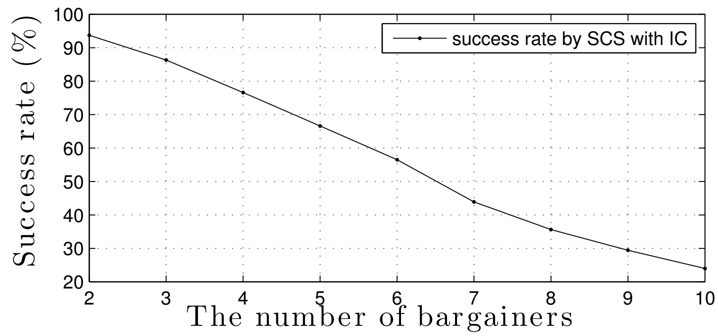


Figure 4: Agreement quality

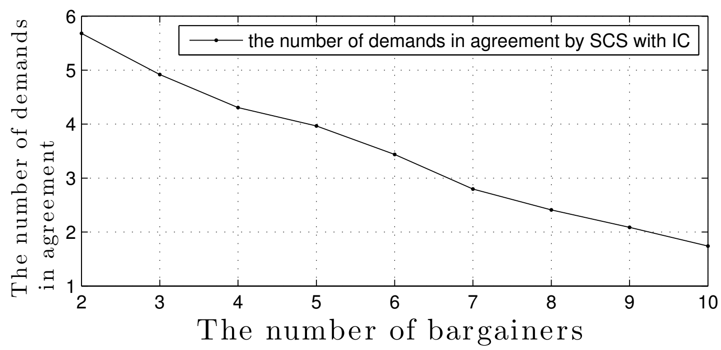
contradict other bargainers' demands in earlier stages of the bargaining, and therefore the probability of reaching an agreement will increase.

From Fig. 3, we can see that no matter how many conflicting demands in the bargaining, the bargainers need to bargain for averagely more rounds to reach an agreementthe if some of them are risk-seeking. Moreover, the efficiency of the bargaining with both risk-averse bargainers is the highest. It means that the bargaining with risk-averse bargainers runs more efficiently.

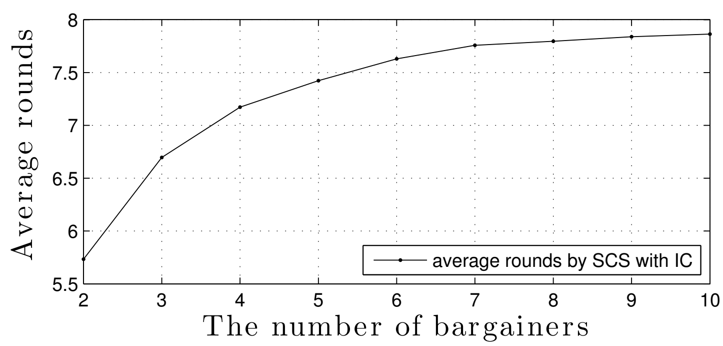
From Fig. 4, we can observe that the remaining demands in the agreement for the bargaining game with risk-averse bargainers are more than that for the bargaining game with risk-seeking bargainers, i.e., the quality of the bargaining with risk-averse bargainers is higher than that of the bargaining with risk-seeking bargainers. This result is also reasonable. In fact, the number of the total demands and the number of the conflicting demands are both fixed, the risk-averse bargainers will put the conflicting demands in the lower preference levels, and then in the earlier stage of the bargaining, the conflicting demands will be removed by concessions of both bargainers. As a result, as many consistent demands as possible will be retained in the agreement. If the bargainers are risk-seeking, they will put the conflicting but desired demands in higher preference levels. This is because bargainers make concessions by giving up their least preferred demands, and thus some inconsistent demands will be removed by other bargainers in earlier stages of the bargaining. Of course, there is a risk: the bargaining will be broken down if the other side also thinks similarly.



(a)



(b)



(c)

Figure 5: The bargaining success-rate and efficiency as well as the agreement quality change with the number of the bargainers, respectively.

5.3. The Number of Bargainers

In this subsection, we will experimentally reveal how the number of the bargainers impacts upon the bargaining solution when other factors of the bargaining remain unchanged. In the following experiment, we randomly generate 10 demands in different preference levels for M bargainer (changing from 2 to 10), and randomly select 4 of them as the conflicting ones for all bargainers. In addition, we generate 2 integrity constraints for each bargaining game.

The experimental results are shown in Fig. 5. From Figs. 5(a) and (b), we can see that the bargaining success-rate and the number of the demands in the agreement decreases when the number of the bargainers increases. And from Fig. 5(c), we can see that the average round needed to reach an agreement in the bargaining increases with the number of the bargainers in the bargaining. These results indicate that if other factors of the bargaining are fixed, the more bargainers in the bargaining, the harder for them to reach an agreement, and the more concession they have to make in order to reach an agreement. In addition, the agreement quality decreases with the increase of the number of the bargainers.

The properties revealed in this section are obvious intuitively. So, by these experiments we can conclude that our model is consistent with human intuitions very well. In other words, our model is effective snd valid.

6. Related Work

The investigation of the bargaining theory diverges into two directions: numerical models and ordinal models. In Nash's seminal paper [20], he defined a numerical model for the bargaining situation, proposed a set of axioms that he thought a solution should satisfy, and established the existence of a unique solution satisfying all the axioms. Nowadays numerical bargaining models have been studying extensively. For example, in [9] a tree structure for the bargaining process is proposed, the fixed point of the bargaining system is studied, and various desirable properties for the solution concept are also analysed.

However, in many real-world bargaining situations, it is very difficult to measure the utility of a bargainer using a numerical scale. Thus, an ordinal bargaining model was proposed by Sapley and Shubik in [25]. They model a bargaining situation in terms of bargainers' preference orderings over possible agreements. From then on, more logic frameworks for bargaining problems have been proposed. Here are some examples. Booth [5] proposed a negotiation model based on multi-agent belief contraction. Zhang et al. [32] introduced the idea of modelling negotiation as a process of mutual belief revision. Meyer et al. [19] discussed the logical properties of the negotiation model based on AGM theory [3]. Chen et al. [6] propose a bargaining procedure to demonstrate how two agents reach an agreement through abductive reasoning. Vo and Li [26] build a logical bargaining model, in which bargainers' beliefs about the bargaining situation is described in propositional logic language and the preference over outcomes is ordinal. Their main contribution is a set of axioms by which the proposed solution can uniquely identified. Another remarkable feature of this model is that their bargaining model integrates concession and argumentation together. Zhan et al. [29, 28, 27] propose a kind of logical bargaining modelS, in which bargainers can change their demand preferences during a bargaining to increase the chance to reach an agreement and use fuzzy rules to calculate the degrees to which a bargainer should change his preference during a bargaining. However, all the logical frameworks mentioned above are constructed without considering the domain constraints in the bargaining, which is studied in our paper.

In the field of automated negotiation, lots of models put constraints into consideration [18], but they are quite different from our work in this paper. For example, the differences between our work and a typical constraint based negotiation model proposed by Luo et al. in [17] are: (i) theirs just negotiates for a single demand, while ours bargains for multiple demands; (ii) their demand is specified a value assigment to all the attributes of the demand (i.e., a product), while our demand is specified by a set of logic proposition formulas; and (iii) they did not prove their solution can be uniquely charactised by a set of properties (i.e., axioms), while we did in this work.

In particular, our work in this paper has extended Zhang's work [30] in several aspects.

- Our model solves the problem of incorporating integrity constraints with bargainers' demands. In Zhang's model [30], a demand of a bargainer can be everything that is related to the bargaining and that the bargainers wants to keep in the final agreement. They do not distinguish the desired constraints or commonsense from the *real* demands. However, our model is more intuitive because it contains the integrity constraints, which are the specifying domain constraints in the bargaining. So, in our model the solution to a bargaining game is constructed based on the hierarchies of demand structures under integrity constraints, while in Zhang's model the solution to a bargaining game is constructed based on the demand structures only.

- We add a logical requirement for the preference ordering over the bargainers' demands to ensure that the preference ordering is rational under a set of integrity constraints. Rather, in Zhang's model, the preference ordering just is subjective.

- Most importantly, in real-life bargaining, the solution usually does not rely on the syntax of the description. However, this cannot hold in Zhang's system, while it is the case in our model as we prove the equivalence property (see Theorem 2). In this case, even the bargainers describe their demands in different forms, the solutions will be the same as long as their desires are essentially the same.

As one of the frameworks for conflicting resolution with integrity constraints, this work also has a close relationship with models of belief merging or database merging with constraints [16, 13]. Following Lin and Mendelzon's work [16] on database merging, Konieczny and Pérez extended the framework of belief merging [14] by adding integrity constraints, which leaded to a new framework of belief merging [13]. Since both bargaining and belief merging require to incorporate information from different sources and thus share some similar properties, for instance, the integrity constraints should be consistent with the outcomes of bargaining or merging. However, the ways of handling information sources are totally different between a merging model and a bargaining model. In merging, the information sources are passive; while in a bargaining, bargainers are initiative, i.e., they can choose different strategies in order to benefit more from the bargaining. In belief merging, sometimes whether an item should be included in the merging outcome relies on how many sources contain this item by the majority rule; while in bargaining, the outcome of a bargaining relies on how firmly the bargainers insist on their demands. In addition, with belief merging, each data source normally does not have a preference over the items in the belief base; while in bargaining, bargainers' preferences over their demands are essential. All these differences have been reflected in the frameworks of belief merging and logical based bargaining.

In game theory, on the one hand, some investigations deal with constraints, but they are not about bargaining games. For example, Zhang et al.

[34] study how constraints influence the outcomes of static games rather than bargaining games. On the other hand, some bargaining games are based on logic, but have not put constraints into considerations. For example, Dunne et al. [7] introduced a bargaining protocol for Boolean games [10, 2]. In a Boolean game, each player has a goal that is represented by a logic formula and each strategy of a player is an action variable which is a component of its goal and others' goals. However, in their model, no constraints are involved and even if we view each player's goal as his demand, it is just like a single demand bargaining model; while ours is a multi-demand bargaing model with constraints.

7. Conclusions

In this paper, we propose a logical model for multi-demand bargaining with integrity constraints and introduce a solution conception to the games of this kind, called a simultaneous concession solution, which satisfies five logical properties and two game theoretical properties. Moreover, we prove that our solution can be characterised uniquely by the five logical properties. In addition, we have done lots experiments to analyse how the number of conflicting demands, bargainers' risk attitude, and bargainer number influence the bargaining success-rate and efficiency as well as the agreement quality. In the future, except simultaneous concession, other bargaining protocols (e.g., various concession strategies in [21]) can be integrated into our logical model of bargaining. Also it is interesting to reveal more game theoretic properties in our logical model of bargaining games.

Acknowledgements

This paper is the significantly enhanced version of our conference paper [12]. And this work was supported by the International Exchange Program Fund of 985 Project and Bairen Plan of Sun Yat-sen University, Raising Program of Major Project of Sun Yat-sen University (No. 1309089), National Natural Science Foundation of China (No. 61173019), MOE Project of Key Research Institute of Humanities and Social Sciences at Universities (No. 13JJD720017) China, and Major Projects of the Ministry of Education China (No. 10JZD0006).

References

[1] Revisions of Knowledge Systems Using Epistemic Entrenchment, 1988.

[2] T. Ågotnes, P. Harrenstein, W. van der Hoek, M. Wooldridge, Verifiable equilibria in Boolean games, in: Proceedings of the 23rd International Joint Conference on Artificial Intelligence, 2013, pp. 689–695.

[3] C.E. Alchourrón, P. Grdenfors, D. Makinson, On the logic of theory change: Partial meet contraction and revision functions, Journal of Symbolic Logic 50 (2013) 510–530.

[4] K. Binmore, A. Rubinstein, A. Wolinsky, The Nash bargaining solution in economic modelling, The RAND Journal of Economics (1986) 176–188.

[5] R. Booth, Social contraction and belief negotiation, in: Proceedings of the 8th International Conference on Principles and Knowledge Representation and Reasoning, volume 2, pp. 375–384.

[6] W. Chen, D. Zhang, M. Wu, A sequential model for reasoning about bargaining in logic programs, in: Logic Programming and Nonmonotonic Reasoning, Springer, 2013, pp. 239–244.

[7] P.E. Dunne, W. van der Hoek, S. Kraus, M. Wooldridge, Cooperative Boolean games, in: Proceedings of the 7th International Joint Conference on Autonomous agents and multiagent systems, volume 2, 2008, pp. 1015–1022.

[8] S.S. Fatima, M. Wooldridge, N.R. Jennings, An agenda-based framework for multi-issue negotiation, Artificial Intelligence 152 (2004) 1–45.

[9] A. Ghosh, S. Kale, K. Lang, B. Moseley, Bargaining for revenue shares on tree trading networks, arXiv preprint arXiv:1304.5822 (2013).

[10] P. Harrenstein, W. van der Hoek, J.J. Meyer, C. Witteveen, Boolean games, in: Proceedings of the 8th Conference on Theoretical Aspects of Rationality and Knowledge, 2001, pp. 287–298.

[11] N.R. Jennings, P. Faratin, A.R. Lomuscio, S. Parsons, M.J. Wooldridge, C. Sierra, Automated negotiation: Prospects, methods and challenges, Group Decision and Negotiation 10 (2001) 199–215.

[12] X. Jing, D. Zhang, X. Luo, A logic framework of bargaining with integrity constraints, in: AI 2013: Advances in Artificial Intelligence, volume 8272 of *Lecture Notes in Computer Science*, 2013, pp. 1–13.

[13] S. Konieczny, R.P. Pérez, Merging information under constraints: A logical framework, Journal of Logic and Computation 12 (2002) 773–808.

[14] S. Konieczny, R. Pino-Pérez, On the logic of merging, in: Proceedings of the 6th International Conference on Principles of Knowledge Representation and Reasoning, Morgan Kaufmann, 1998, pp. 488–498.

[15] S. Kraus, K. Sycara, A. Evenchik, Reaching agreements through argumentation: A logical model and implementation, Artificial Intelligence 104 (1998) 1–69.

[16] J. Lin, A.O. Mendelzon, Merging databases under constraints, International Journal of Cooperative Information Systems 7 (1998) 55–76.

[17] X. Luo, N.R. Jennings, N. Shadbolt, H.f. Leung, J.H.m. Lee, A fuzzy constraint based model for bilateral, multi-issue negotiations in semi-competitive environments, Artificial Intelligence 148 (2003) 53–102.

[18] X. Luo, C. Miao, N.R. Jennings, M. He, Z. Shen, M. Zhang, KEMNAD: A knowledge engineering methodology for negotiating agent development, Computational Intelligence 28 (2012) 51–105.

[19] T.A. Meyer, N.Y. Foo, R. Kwok, D. Zhang, Logical foundations of negotiation: Strategies and preferences., in: Proceedings pf the 9th International Conference on the Principles of Knowledge Representation and Reasoning, 2004, pp. 311–318.

[20] J.F. Nash Jr, The bargaining problem, Econometrica 18 (1950) 155–162.

[21] L. Pan, X. Luo, M. Meng, C. Maio, M. He, X. Guo, A two-stage win-win multiattribute negotiation model: optimazationa and then concession, Computational Intelligence 29 (2013) 577–626.

[22] S. Parsons, C. Sierra, N.R. Jennings, Agents that reason and negotiate by arguing, Journal of Logic and Computation 8 (1998) 261–292.

[23] H. Raiffa, The Art and Science of Negotiation, Harvard University Press, 1982.

[24] A.E. Roth, Axiomatic models of bargaining, Technical Report, David K. Levine, 1979.

[25] M. Shubik, Game theory in the social sciences: Concepts and solutions, MIT Press, 2006.

[26] Q.B. Vo, M. Li, From axiomatic to strategic models of bargaining with logical beliefs and goals, in: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems, volume 1, 2012, pp. 525–532.

[27] J. Zhan, X. Luo, C. Feng, W. Ma, A fuzzy logic based bargaining model in discrete domains: Axiom, elicitation and property, in: Proceedings of the 2014 IEEE International Conference on Fuzzy Systems.

[28] J. Zhan, X. Luo, W. Ma, Y. Zhang, A multi-demand adaptive bargaining based on fuzzy logic, in: Proceedings of the 6th International Conference on Agents and Artificial Intelligence, pp. 577–585.

[29] J. Zhan, X. Luo, K.M. Sim, C. Feng, Y. Zhang, A fuzzy logic based model of a bargaining game, in: Knowledge Science, Engineering and Management, volume 8041 of *Lecture Notes in Artificial Intelligence*, Springer, 2013, pp. 387–403.

[30] D. Zhang, A logic-based axiomatic model of bargaining, Artificial Intelligence 174 (2010) 1307–1322.

[31] D. Zhang, N. Foo, Infinitary belief revision, Journal of Philosophical Logic 30 (2001) 525–570.

[32] D. Zhang, N. Foo, T. Meyer, R. Kwok, Negotiation as mutual belief revision, in: Proceedings of the 19th National Conference on Artificial Intelligence, 2004, pp. 317–322.

[33] D. Zhang, Y. Zhang, An ordinal bargaining solution with fixed-point property, Journal of Artificial Intelligence Research 33 (2008) 433–464.

[34] Y. Zhang, X. Luo, H.f. Leung, Fuzzily constrained games, in: Proceedings of the 2013 IEEE/WIC/ACM International Conference on Web Intelligence and Intelligent Agent Technology, pp. 361–368.