Convergency of Learning Process

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Abstract. This paper presents a learning theoretic analysis on stability of learning in light of iterated belief revision. We view a learning process as a sequential belief change procedure. A learning policy is sought to guarantee every learning process leads to a complete knowledge about the world if the newly accepted information is the true fact on the world. The policy allows an agent to abandon the knowledge it has learned but requires a relatively moderate attitude to new information. It is shown that if new information is not always accepted in an extremely skeptical attitude and the changes of belief degrees follow the criterion of minimal change, any learning process for learning truth will converge to a complete knowledge state.
Keywords: belief revision, iterated belief change, learning theory

1 Introduction

Theories of belief change address the problem of how an agent revises its beliefs when it learns new information about the world. Such theories have been received considerable concern in both philosophy and artificial intelligence partially because they provide formal mechanisms for modelling the evolution of knowledge of human being and the one of knowledge bases. The work on iterated belief change has even reinforced such mechanisms with considering belief and knowledge evolution as a continuous learning process. Almost every aspect of the specification of iterated belief change, including axiomatization, semantic models and computational models, has been investigated in the literature. ([Spohn 1988][Boutilier 1993][Nayak 1994][Lehmann 1995][Williams 1995][Zhang 1995][Darwiche and Pearl 1997][Kelly 1998]). However, it is largely unexplored in either belief revision or learning theory literature that when the processes of sequential belief change is reliable and guaranteed to stabilize to true.

Formally, let Γ be the initial belief state of an agent and $\{A_i\}_{i=1}^{\infty}$ a sequence of pieces of information received by the agent. A learning process over Γ_0 and $\{A_i\}_{i=1}^{\infty}$ can then be specified by the sequence $\{\Gamma_i\}_{i=0}^{\infty}$, which is defined recursively as follows:

$\Gamma_0 = \Gamma$

$\Gamma_i = \Gamma_{i-1} * A_i$

where $*$ is an iterated belief revision operator.

The main question before us is what kind of learning policy a rational agent should hold to guarantee the reliability and stability of learning processes. Some investigations towards the problem have been done in the literature. [Zhang *el al* 1997] [Zhang and Foo 2001] explored the convergency of belief states in the settings when the newly received information comes from an infinite set of propositions. An assumption, called Limit Postulate, has been made to guarantee that the sequence of belief change by the finite subsets of an infinite set of propositions converges to the belief change by the infinite set. [Kelly 1998] contributed a more profound analysis on the learning powers of some concrete iterated belief revision mothods proposed by [Spohn 1988] [Boutilier 1993] [Nayak 1994] [Goldszmidt and Pearl 1994] and [Darwiche and Pearl 1997]. Some interesting results have been obtained through the analysis that different learning policies (minimal change) induced by different belief revision methods fall to different hierarchies of learning power, an objective measure of reliability of learning process.

This paper focus on the analysis of stability of learning process based on a flexible minimal change policy. We shall assume that every belief change is reliable in the sense that each piece of new information received by an agent is a true fact about the world. We are interested in the question that what kind of learning policy should be held by a rational agent to guarantee its belief state to lead to a complete knowledge state about the world. In other words, if all pieces of the newly received information are the truth of the world, whether a learning process converges to a complete knowledge state of the world. We prove that if an agent is not extremely stubborn on its old beliefs, and updates of epistemic state follow a rational procedure of minimal changes, the process of learning the truth will converge to the set of all the truth.

Throughout this paper, we consider the first-order language $\mathcal{L}$ as the object language. L is the set of all sentences in $\mathcal{L}$. We denote individual sentences by A, B, or C, and denote sets of sentences by Γ, Δ etc. We shall assume that the underlying logic includes the classical first-order logic with the standard interpretation. The notation $\vdash$ means the classical first-order derivability and Cn the corresponding closure operator, i.e.,

$$A \in Cn(\Gamma) \text{ if and only if } \Gamma \vdash A$$

2 Minimal change policies of iterated belief revision

AGM theory specifies the change of belief state by a rational agent when the agent received a piece of new information. However, it has been pointed out by several authors that the AGM theory is not sufficient to specify a sequential process of belief change due to it ignored the change of agent's epistemic state after each change of belief state([Spohn 1988][Boutilier 1993][Nayak 1994] [Goldszmidt and Pearl 1994][Lehmann 1995][Williams 1995][Zhang 1995][Darwiche and Pearl 1997]. It is true that such a update of epistemic state is mainly determined by the agent's subjective estimation on its new belief state. However, some assumptions can still be made to demonstrate how a rational agent would change its epistemic state. A common assumption in the light is that the ordering information, such as epistemic entrenchment or strength of belief, which guide the revision operation of agent, should be preserved as much as possible. Such an assumption is called *minimal change of belief degrees* in light of information economics.

[Boutilier 1993] proposed a *natural* method of epistemic state modification based on the idea that new information is always accepted in the lowest degree of epistemic entrenchment while preserves the entrenchment ordering as much as possible. On the assumption, it was shown that

$$((K * A_1) * \cdots) * A_n = K * A_1 \wedge \cdots \wedge A_k \wedge A_n$$

where A_k $(k < n)$ is the most recent formula such that $((K * A_1) * \cdots) * A_k$ is consistent with A_n. [Nayak 1994] and [Zhang 1995], however, took the opposite assumption that the newly arrived information always takes priority over the existing beliefs. Based on the assumption, it can be shown that $((K * A_1) * \cdots) * A_n = K * (A_1 \wedge \cdots \wedge A_n)$ if $\{A_1, \cdots, A_n\}$ is consistent; otherwise, $((K * A_1) * \cdots) * A_n = Cn(A_k \wedge \cdots \wedge A_n)$ where k is the minimum such that $\{A_k, \cdots, A_n\}$ is consistent.

It is not difficult to find some counterintuitive examples in both systems(see [Darwiche and Pearl 1997]). This is because that the former takes an extremely skeptical view of new information, while the latter holds the other extreme. More deliberate approaches have proposed also to take account of agent's subjective estimation on its new beliefs (insert the new beliefs into the ordering of belief state and adjust the ordering in order to satisfy the criteria of epistemic entrenchment[Williams 1995]). It could be expected that a flexible minimal change policy over belief revision procedures, which imposes only loose constraints on the change of epistemic state, would be able to guarantee convergency of learning processes even we are lack of subjective estimation on newly arrived information from the epistemic agent.

Let us consider the situation when all the information an agent received is always true facts about the world. Does the agent's belief state approach eventually to a true understanding on the world? In other words, if τ denotes all pieces of the truth about the world, and the agent learns them step by step, does the agent will have all the truth? It is easy to see that this is true for Nayak's approach, but may be failed with Boutilier's assumption because in his system when a new piece of information happens to be inconsistent with all previous belief states of the agent (say the agent is extremely stubborn on some totally wrong ideas), the agent would give up the facts which contradict the wrong ideas even though they could have been reluctantly accepted in previous revision steps. Therefore, the road to truth needs a moderate attitude of new information. In fact, we will prove that if the new information is not always accepted in an extremely skeptical attitude and changes of belief degrees follow the criterion of minimal changes, the process of learning the truth will converge to the set of all the truth.

3 Total-ordered partitions and belief revision operators

A belief revision occurs when a new piece of information that is inconsistent with the present belief state of an agent is added to the state. In this process, some old beliefs will be abandoned and others retained. Such selection is generally assumed under the guidance of some ordering information, typically Spohn's plausibility on possible worlds and AGM's epistemic entrenchment on sentences (see [Gärdenfors 1988]). For the purpose of this paper, we will exploit a kind of ordering structure, called total-ordered partition, introduced in [Zhang and Foo 2001], which is similar to but weaker than the epistemic entrenchment ordering. The underlying intuition is that an agent could organize all its beliefs into several groups in terms of the degrees in which it believes them. Sentences in the same group are of nearly equal degree of belief. All the groups are then arranged in a total ordering or a well ordering.

Definition 1. [Zhang and Foo 2001] Let Γ be a set of sentences, $\mathcal{P}$ a partition of Γ, and $<$ a total ordering relation on $\mathcal{P}$. The triple $\Sigma = (\Gamma, \mathcal{P}, <)$ is called *a total-ordered partition(TOP) of* Γ. If $<$ is a well ordering on $\mathcal{P}$, Σ is called *a well-ordered partition (WOP) of* Γ.

For any $P \in \mathcal{P}$ and $A \in P$, P is called the rank of A, denoted by $r(A)$. Sentence in lower rank is considered with higher degree of belief. It is easy to see that the rank gives a complete pre-order on Γ, so in a sense a total-ordered partition is equivalent to the degree of epistemic relevance in [Nebel 1992].

Following [Nebel 1992], we define a notation $\Downarrow$ as follows: for any set Γ of sentences and any sentence A, $\Delta \in \Gamma \Downarrow A$ if and only if

$$\Delta \text{ is a subset of } \Gamma \text{ and } \Delta = \bigcup_{P \in \mathcal{P}} \Delta_P,$$

where for any $P \in \mathcal{P}$, Δ_P is a maximal subset of P such that $(\bigcup_{Q \leq P} \Delta_Q) \cup \{\neg A\}$ is consistent.

In fact, $\Gamma \Downarrow A$ is just a specialization of the maximal consistent subset family $\Gamma \perp A$ (see [Gärdenfor 1988]). The lower the rank of a sentence, the more priority it is chosen.

With this notation, we could define the following revision operators. Let Γ be a set of sentences and Σ a total-ordered partition of Γ.

- *Belief revision* of Γ by A:
  $\Gamma *_1 A = \bigcap(\Gamma \Downarrow \neg A) + A$;
- *Belief base revision* of Γ by A:
  $\Gamma *_2 A = \bigcap_{\Delta \in \Gamma \Downarrow \neg A} Cn(\Delta) + A$;
- *Reconstruction* of Γ by A: $\Gamma *_3 A =$
  $(\bigcap(\Gamma \Downarrow \neg A)) \cup (\{A\} \setminus Cn(\bigcap(\Gamma \Downarrow \neg A)))$.

The operator $*_1$ comes from [Zhang and Foo 2001][3]. It has been proved that if Γ is deductively closed and Σ is a nice-ordered partition of Γ [4], $*_1$ satisfies all the AGM's postulates for revision.

The operator $*_2$ comes from [Nebel 1992]. If Γ is deductively closed, it has been proved that $*_2$ satisfies all the AGM's postulates for revision but $(K * 8)$.

The operator $*_3$ comes form [Zhang and Li 1998]. The differences between $*_2$ and $*_3$ are:

[3] It is easy to show that if Σ is a well-ordered partition, this revision operator is equivalent to the one in [Williams 1995].

[4] Σ is a nice-ordered partition of Γ if Σ is a total-ordered partition of Γ and satisfies:
If $A_1, \cdots, A_n \vdash B$, $\sup\{r(A_1), \cdots, r(A_n)\} \leq r(B)$ (See [Zhang and Foo 2001])

1. $\Gamma *_3 A$ need not to be deductively closed;
2. $A \in \Gamma *_3 A$ need not to be true.

In this paper, we refer the revision operator $*$ instead of in particular to some special one but to any revision function which can be determined by a total-ordering partition[5] and satisfies the following properties:

(A1) $A \in Cn(\Gamma * A)$;
(A2) $\Gamma * A \in Cn(\Gamma \cup \{A\})$;
(A3) $\Gamma * A$ is consistent iff $\nvdash \neg A$;
(A4) $A \vdash\dashv B$ implies $Cn(\Gamma * A) = Cn(\Gamma * B)$.

where Γ and $\Gamma * A$ could be non-closed set of sentences. It is obvious that $*_1, *_2, *_3$ satisfy (A1)-(A4) and any AGM revision operator belongs to this category.

4 Minimal change of belief degree and learning process

In the process of belief revision of an agent, beliefs change not only in numbers but also in degrees of belief. In general, an epistemic agent would have an estimation of belief degrees for its beliefs. Such estimation in each revision will influence the result of the next revision. Although belief degree may be the agent's subjective evaluation, a rational assumption seems to be that change of belief degree should be minimal in the case of absence of subjective information from the epistemic agent. In other words, the ordering on the original belief state should be preserved as much as possible. As we have mentioned, such a principle is called *minimal change of belief degree*, which can be specified by the following definition:

Definition 2. Let $\Sigma = (\Gamma, \mathcal{P}, <)$ be a well-ordered partition (WOP) of Γ, η be the order-type of $\mathcal{P}$. For any sentence A and an ordinal α, define a WOP $\Sigma_A^\Gamma(\alpha) = (\Gamma * A, \mathcal{P}_A^\Gamma, <_A^\Gamma)$ of $\Gamma * A$ as follows:

For any $\beta < \max\{\alpha + 1, \eta\}$, let

$$P'_\beta = \begin{cases} (P_\beta \cup \{A\}) \cap (\Gamma * A), & \text{if } \beta = \alpha; \\ P_\beta \cap (\Gamma * A), & \text{otherwise.} \end{cases}$$

$$P''_\beta = (Cn(P'_\beta) \cap (\Gamma * A)) \setminus \bigcup_{\gamma < \beta} P''_\gamma$$

Let $\mathcal{P}_\mathrm{A}^\Gamma = \{\mathrm{P}''_\beta : \beta < \max\{\alpha + 1, \eta\}\}$. For any P_β, $P_\gamma \in \mathcal{P}_A^\Gamma$, define that

$$P_\beta <_A^\Gamma P_\gamma \quad \text{if and only if} \quad \beta < \gamma$$

$\Sigma_A^\Gamma(\alpha)$ is called *the minimal change of belief degrees for Σ w.r.t. A and α*.[6]

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[5] So the outcomes in sequent sections is suitable at least for these three kinds of revision operation.

[6] We do not consider the case that $A \in \Gamma$. In this case, a rational restriction is the ordering keeps unchanged.

It is not difficult to see that $\Sigma_A^\Gamma(\alpha)$ specifies such a well-ordered partition that the new piece of information A is accepted in the degree α and the old beliefs preserve the original ordering.

Now let's consider a learning process of an epistemic agent. Suppose Γ_0 is the initial belief state of the agent and Σ_0 is a WOP of Γ_0. Whenever the agent learns a piece of new information, its belief state will evolve into a new one. We call such evolutionary process of belief state a learning process of the agent. More precisely, we have the following definition.

Definition 3. Let Γ_0 be a set of sentences with a WOP Σ_0. Let $\{A_i\}_{i=1}^n$ be a sequence of sentences, $\{\alpha_i\}_{i=1}^n$ a sequence of ordinal numbers. Define recursively a sequence of sets $\{\Gamma_i\}_{i=1}^n$ and a sequence of well-ordered partitions $\{\Sigma_i\}_{i=1}^n$ as follows:

i). $\Gamma_i = \Gamma_{i-1} * A_i$, where the revision operation is based on the well-ordered partition Σ_{i-1} of Γ_{i-1};
ii). Σ_i is the minimal change of belief degree for Σ_{i-1} w.r.t. A_i and α_i.

$\{\Gamma_i\}_{i=0}^n$ is called *the learning process w.r.t.* $\{A_i\}_{i=1}^n$ *and* $\{\alpha_i\}_{i=1}^n$ *started from* Γ_0.

5 Convergency of learning process

Suppose a learning process started from a consistent belief state. In stead of arbitrary import of information, we suppose that all the information the agent learned are the truth or knowledge about the world. It is obvious that an agent is unlikely to keep all the learned truth in its belief states forever even though new information is always let in when it is being learnt and all the new information is consistent. However, we do not intend to impose a radical policy on it to enforce it holding all the learned things. We will allow an agent to abandon some truths it ever learned, but expect that it does not take an extremely skeptical view of new information so that every piece of the truth could be accepted. What we are interested in is whether such a moderate learning policy could promise a learning process converges. To this end, let us see a general result on the convergency of learning process.

Theorem 1. *Let M be a model of $\mathcal{L}$. $\tau_M = \{A \in L : M \models A\}$. Γ_0 is a set of sentences with a WOP Σ_0. For any sequence of ordinals $\{\alpha_i\}_{i=1}^\infty$ and an enumeration $\{A_i\}_{i=1}^\infty$ of τ_M. If $\{\Gamma_i\}_{i=1}^\infty$ is a learning process w.r.t. $\{A_i\}_{i=1}^\infty$ and $\{\alpha_i\}_{i=1}^\infty$ started from Γ_0, then*

$$\underline{\lim}_{n\to\infty} Cn(\Gamma_n) \subseteq \tau_M \subseteq \overline{\lim}_{n\to\infty} Cn(\Gamma_n) \tag{1}$$

specially, if $\Gamma_0 \setminus \tau_M$ is finite, then

$$\lim_{n\to\infty} Cn(\Gamma_n) = \tau_M \tag{2}$$

Proof: First it is easy to see that for any $i > 0$, Γ_i is consistent because A_i is consistent.

(a). Suppose that $A \in \tau_M$. For there are infinite sentences in τ_M being equivalent to A, let these sentences form a subsequence $\{A_{k_j}\}_{j=1}^\infty$ of $\{A_i\}_{i=1}^\infty$. By the construction of learning process, for any $j \geq 1$, we have $A_{k_j} \in Cn(\Gamma_{k_j})$, or $A \in Cn(\Gamma_{k_j})$, which means $A \in \overline{\lim_{n\to\infty}} Cn(\Gamma_n)$. Thus $\tau_M \subseteq \overline{\lim_{n\to\infty}} Cn(\Gamma_n)$.

(b). Suppose that $A \in \underline{\lim_{n\to\infty}} Cn(\Gamma_n)$. Then there is a number n_0 such that $A \in Cn(\Gamma_n)$ for any $n \geq n_0$. If $A \notin \tau_M$, then $\neg A \in \tau_M$. By the proof of (a), there is a subsequence $\{A_{k_j}\}_{j=1}^{\infty}$ of $\{A_i\}_{i=1}^{\infty}$ such that A$_{k_j} \mapstochar\!\dashv \neg A$ and $A_{k_j} \in Cn(\Gamma_{k_j})$ for any $j \geq 1$. Thus there exists $k_{j_0} \geq n_0$ such that $\neg A \in Cn(\Gamma_{k_{j_0}})$. But $A \in Cn(\Gamma_{k_{j_0}})$, which contradicts to the consistency of $\Gamma_{k_{j_0}}$. Thus $A \in \tau_M$, that is, $\underline{\lim_{n\to\infty}} Cn(\Gamma_n) \subseteq \tau_M$.

(c). If $\Gamma_0 \setminus \tau_M$ is finite. Let $\Gamma_0 \setminus \tau_M = \{B_0, \cdots, B_m\}$. Then $\{\neg B_0, \cdots, \neg B_m\} \subseteq \tau_M$. Therefore there exists n_0 such that $A_{n_0} \mapstochar\!\dashv \neg B_0 \wedge \cdots \wedge \neg B_m$. By the properties of belief revision and the definition of learning process, $A_{n_0} \in Cn(\Gamma_{n_0})$. Because $\Gamma_{n_0} \subseteq \Gamma_0 \cup \tau_M$ and Γ_{n_0} is consistent, we have $\Gamma_{n_0} \subseteq \tau_M$. Then, by the definitions of learning process and revision, we conclude that $\{\Gamma_n\}_{n=n_0}^{\infty}$ is a monotonic increasing sequence, which means $\{Cn(\Gamma_n)\}_{n=0}^{\infty}$ converges. Therefore, equation (1) implies (2). $\square$

In this theorem, M acts as an ideal model in which every satisfied statement is truth. This theorem then shows that any learning process started from a finite set will converge no matter how new information is accepted. This result, however, can not be generalized to the case that the starting point is an infinite set. In fact, a learning process would diverge even though new information is always contained in the new belief state, and all the new information is consistent.

We consider that the following additional assumption is rational:

1. The new information is not always accepted with the extremely skeptical attitude and any new information has opportunity to be accepted with relative high degrees of belief.

2. When a piece of information is accepted several times, its relative belief degrees in the learning process should not decrease.

Under this consideration, we give the following definition of rational learning processes:

Definition 4. Suppose the definitions of M, τ_M and Γ_0 as Theorem 1. $\{A_i\}_{i=1}$ numerates the set τ_M and $\{\alpha_i\}_{i=1}^{\infty}$ is a sequence of natural numbers. $\{\Gamma_i\}_{i=1}^{\infty}$ is a learning process w.r.t. $\{A_i\}_{i=1}^{\infty}$ and $\{\alpha_i\}_{i=1}^{\infty}$ started from Γ_0 and $\{\Sigma_i\}_{i=0}^{\infty}$ is a WOP of $\{\Gamma_i\}_{i=0}^{\infty}$. If the following conditions hold:

i). For any $A \in \tau_M$, there exists number n_0 such that $A_{n_0} \mapstochar\!\dashv A$ and $\{B \in \Gamma_{n_0} \setminus Cn(\phi) : r^{\Sigma_{n_0}}(B) \leq r^{\Sigma_{n_0}}(A_{n_0})\}$ is a finite set[7];

ii). For any $i, j \geq 1$, if $i < j$ and $A_i \mapstochar\!\dashv A_j$,

$$\forall B \in \Gamma_i \cap \Gamma_j (r^{\Sigma_j}(B) \leq r^{\Sigma_j}(A_j) \to r^{\Sigma_i}(B) \leq r^{\Sigma_i}(A_j))$$

Then $\{\Gamma_i\}_{i=1}^{\infty}$ is called *a rational learning process* w.r.t. $\{A_i\}_{i=1}^{\infty}$ and $\{\alpha_i\}_{i=1}^{\infty}$ started from Γ_0.

[7] $r^{\Sigma}(A)$ denotes the rank of A under the partition Σ.

Condition i) means any new information has opportunity to be accepted in a relatively high belief degree (the number of sentences which are not a tautology and their degrees of belief are not lower than this piece of information is finite). Condition ii) shows that when a piece of information is learned more than once, the relative belief degree in which it is accepted should not be lower than those at last times. Then we have

Theorem 2. *Any rational learning process converges and*

$$\lim_{n\to\infty} Cn(\Gamma_n) = \tau_M$$

Proof: We split the proof into the following three steps:

(a). We prove that for any $A \in \tau_M$, there is a natural number N such that $A_N \vdash\dashv A$ and $\{B \in \Gamma_N \backslash Cn(\phi) : r^{\Sigma_N}(B) \leq r^{\Sigma_N}(A_N)\} \subseteq \tau_M$.

In fact, the condition i) implies that there exists n_0 such that $A_{n_0} \vdash\dashv A$ and $\Delta = \{B \in \Gamma_{n_0} \backslash Cn(\phi) : r^{\Sigma_{n_0}}(B) \leq r^{\Sigma_{n_0}}(A_{n_0})\}$ is finite. If $\Delta \setminus \tau_M$ is empty, or $\Delta \subseteq \tau_M$, the proof is ready provided let $N = n_0$. For the case that $\Delta \setminus \tau_M$ is nonempty, let $\Delta \setminus \tau_M = \{C_1, \cdots, C_m\}$, then $\neg C_1 \wedge \cdots \wedge \neg C_m \in \tau_M$, thus there is a number $n_1 \geq n_0$ such that $A_{n_1} \vdash\dashv \neg C_1 \wedge \cdots \wedge \neg C_m$. According to the construction of learning process, $A_{n_1} \in Cn(\Gamma_{n_1})$. Since Γ_{n_1} is consistent, $(\Delta \setminus \tau_M) \cap \Gamma_{n_1} = \phi$. On the other hand, again by the construction of learning process, the sequence $\{\Gamma_n \setminus \tau_M\}_{n=0}^{\infty}$ decreases monotonically, therefore,

$$\forall n \geq n_1((\Delta \setminus \tau_M) \cap \Gamma_n = \phi) \tag{3}$$

Since there are infinite sentences in τ_M which are logically equivalent to A, there exists a number $N \geq n_1$ such that $A_N \vdash\dashv A$. Let $\Delta' = \{B \in \Gamma_N \backslash Cn(\phi) : r^{\Gamma_N}(B) \leq r^{\Gamma_N}(A_N)\}$. With the condition ii) of the rational learning process, we obtain that

$$\Delta' \setminus \tau_M \subseteq \Delta \setminus \tau_M$$

By the expression (3), we have $(\Delta \setminus \tau_M) \cap \Gamma_N = \phi$, thus $(\Delta' \setminus \tau_M) \cap \Gamma_N = \phi$. Note that $\Delta' \subseteq \Gamma_N$, so $\Delta' \setminus \tau_M = \phi$, that is, $\Delta' \subseteq \tau_M$.

(b). Assume that $A \in \tau_M$. By (a), there exists a natural number N such that $A_N \vdash\dashv A$ and $\Delta = \{B \in \Gamma_N \backslash Cn(\phi) : r^{\Sigma_N}(B) \leq r^{\Sigma_N}(A_N)\} \subseteq \tau_M$. Hence $\Delta \cup \{A_n\}$ is consistent for any $n \geq N$. According to the construction of learning process, $\Delta \subseteq \Gamma_n$. Specially, $A_N \in \Gamma_n$, or $A \in Cn(\Gamma_n)$. Thus $A \in \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$. So we have $\tau_M \subseteq \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$.

(c). Suppose that $A \in \overline{\lim}_{n\to\infty} Cn(\Gamma_n)$. If $A \notin \tau_M$, $\neg A \in \tau_M$. By (b), $\neg A \in \underline{\lim}_{n\to\infty} Cn(\Gamma_n)$. Thus there is a number n_0 such that $A \wedge \neg A \in Cn(\Gamma_{n_0})$, which contradicts to the consistency of Γ_{n_0}. Therefore, $\overline{\lim}_{n\to\infty} Cn(\Gamma_n) \subseteq \tau_M$. □

This theorem says that if all the information an agent accepted is the truth about the world, its belief state will converge to the set of all pieces of the truth provided its learning process is rational. Note that the criterion of minimal changes of belief degree and the rationality of learning processes are sufficient but not necessary for the convergency of learning processes. It is not difficult to entail some more loose conditions of the convergency from the proof of the theorem.

6 Conclusion

We have investigated a learning policy to accept new information for a rational epistemic agent. Such a policy allows an agent to abandon the knowledge it has learned but requires a relatively moderate attitude to new information. We have proved that if the new information is not always accepted in an extremely skeptical attitude and changes of belief degrees follow the criterion of minimal changes, the process of learning the truth converges to the set of all the truth.

There have been several works on the analysis of learning theory by applying belief revision methods. [Eric Martin and Osherson 2000] offered a model of inductive inquiry on the basis of belief revision operation. [Kelly 1998] presented an approach to analyze the reliability of learning process based on the existing iterated belief revision operators. Different from Kelly's, our analysis on learning process does not depend on any particular iterated belief change operator. In fact, our assumption on the change of epistemic state is the loosest one among the assumptions of the existing iterated belief revision operations. However, our assumption is not a necessary condition to guarantee the convergency of learning process. It is left open how to find a sufficient and necessary condition for the convergency of learning process.

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