Interpolation Properties of Action Logic: Lazy-formalization to the Frame Problem

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ABSTRACT

This paper makes a contribution to the meta-theory of reasoning about action. We present two interpolation properties of action logic. We show that the frame axioms which are required for answering a query involve only the objects which are relevant to the query and action description. Moreover, if the action description is expressed by normal form, the required frame axioms depend on only the query itself. Therefore the frame problem may be mitigated by localizing descriptions and postponing the listing of frame axioms till a query occurs. This offers a pragmatic solution to the frame problem. This solution does not rest on any meta-hypotheses most existing solutions to the frame problem rely on.

Keywords: frame problem, reasoning about action, dynamic logic.

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Abstract. This paper makes a contribution to the meta-theory of reasoning about action. We present two interpolation properties of action logic. We show that the frame axioms which are required for answering a query involve only the objects which are relevant to the query and action description. Moreover, if the action description is expressed by normal form, the required frame axioms depend on only the query itself. Therefore the frame problem may be mitigated by localizing descriptions and postponing the listing of frame axioms till a query occurs. This offers a pragmatic solution to the frame problem. This solution does not rest on any meta-hypotheses most existing solutions to the frame problem rely on.

1 Introduction

The theory of action and change has been a focus of AI for over twenty years. Most fundamental problems in this area, such as the frame problem, ramification problem, and qualification problem, have been widely investigated with varying degrees of success. The time has come to analyze, compare and systematize these formalisms and solutions in order to obtain a more complete and solid theory of action. This paper makes a contribution to the meta-theory in order to fill the gap between action logic and meta-hypotheses which most exiting solutions to the frame problem rely on.

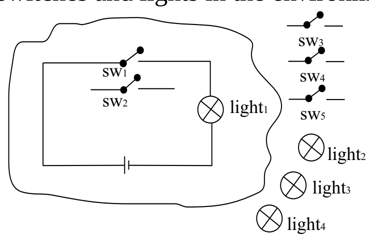
As a central issue in reasoning about action, the frame problem arises upon the assumption that the specification of a system (and its underlying reasoning mechanism) should be strong enough to answer any query by direct inference. With the assumption, a dilemma appears inevitable. On the one hand, most actions have only local effects. So, if one has to explicitly mention all the things that are unaffected, the list (frame axioms) will usually be unmanageably large; On the other hand, if soundness and completeness are desired for the reasoning system, not a single frame axiom can be omitted. Many solutions to the problem have been proposed. These can be grouped into two categories: *monotonic* and *nonmonotonic*.

Monotonic approaches seek automatic procedures to generate frame axioms from effect axioms based on some meta-hypothesis, such as *Explanation Closure Assumption* or *Causal Completeness Assumption*[19][22][20].

Nonmonotonic approaches attempt to introduce new inference mechanisms based on meta-hypothesis such as the *common sense law of inertia* or *minimization*, to capture defaults about the effects of actions implied by frame axioms [9][1][23] [7].

It has been pointed out by several authors that all these meta-hypotheses are essentially equivalent despite the difference in their appearance [13] [14] [15] [6]. Because of their reliance on meta-hypothesis, these theories are apt to overcommit. For instance, when the effect of an action on a fluent is unknown, most formalisms of action would assume, indeed assert, that there is no effect at all but not flag such an assumption. Another price for accommodating such kind of meta-hypotheses is that the associated action logics become more complicated and may lose the completeness of logical inference. This paper aims to introduce an approach to the frame problem which releases the underlying action logic from meta-hypotheses while preserves the soundness and completeness of the logic. To see the possibility, let us consider a simple example.

The following figure shows a circuit in which only switch 1 is connected to the circuit. There are also some other switches and lights in the environment.



We describe this circuit from two points of view: engineering and AI:

1. Engineering description:

"*Turning on Switch 1 causes the light to be on, turning it off causes the light to be off. The action with Switch 2 does not change the state of the light.*"

2. AI description:

"*Turning on Switch 1 causes the light to be on, turning it off causes the light to be off. Any other action does not affect the state of the light.*"

Both descriptions seem to work, but the second sentence in the second description sounds a bit unnatural and arbitrary. It expresses *the common sense law of inertia* or *causal completeness assumption*. With this statement, we can answer "No!" confidently when asked "Does the cough of an ET cause the light to be off?". But while most engineers would regard the seriousness of such questions sceptically, they may not simply answer "No". Instead, they may regard it as a hint that an expected or hitherto unknown influence may exist, and therefore answer, "Possibly."

Then why do engineers not worry about the frame problem? The reasons might be the following. On one hand, for an engineering system, the possible future queries about the system can generally be foreseen. Therefore engineers can localize their language so that it involves only relevant components yet is sufficient for specifying the system and expressing possible future queries. On the other hand, they believe that answering a query in the local language might only require local frame axioms. Therefore the number of frame axioms will mainly depend on the size of the local language.

In this example, if all the possible queries are only concerned with sw_1, sw_2 and $light_1$, the frame axioms required for answering these queries can be restricted to the actions and fluents which are only relevant to these elements. Even though the other switches and lights in the environment are concerned, the size of the frame axioms is still manageable. In this sense, the full-blown frame problem can be avoided. However, this localization approach relies again on another meta-hypothesis: *local queries requires only local frame axioms*. More precisely, let Σ and φ be a set of effect axioms of actions in a system and a query about the system respectively, expressed in an action language $\mathcal{L}$.

Suppose that Δ_φ is the set of frame axioms which is required for answering the query. That is,

$$\Sigma \cup \Delta_\varphi \models_{AL} \varphi \text{ (or } \neg\varphi)$$

where $\models_{AL}$ is the entailment of underlying action logic. Then the assumption we made is that all the frame axioms (Δ_φ) required for the inference are expressible in the sub-language of $\mathcal{L}$ which only involves symbols in Σ and φ. This reminds us the Craig Interpolation Theorem of classical first-order logic, which says that if $\varphi \models \psi$, then there exists a sentence θ such that $\varphi \models \theta$, $\theta \models \psi$ and all the symbols which occur in θ occur in φ and ψ (see [3]). In fact, if the frame axioms can be any formula and $\models_{AL}$ has the similar interpolation property, the assumption mentioned above will be implied by the interpolation theorem.

In this paper we present two interpolation properties of action logic to verify the assumption. Since local queries require only local frame axioms and a query generally involves only few objects, the frame axioms required in a reasoning can be reduced to a reasonable size. Therefore we can simply avoid the frame problem by postponing the listing of the frame axioms until they are needed. We call this approach *lazy-formalization*, which is similar to the technique of *lazy-evaluation* or *by-need* in automated reasoning and programming. A significant difference between our solution and the others is that our solution does not relies on any meta-hypothesis except the logic itself. It becomes a property of the associated action logic. The hard part is how to verify this property. To this end, we need:

1. A formal framework for expressing and reasoning about actions and their effects, enhanced with interpolation property.

2. A normalized expression of frame axioms in order that we can easily generate frame axioms when needed.

3. A suitable inference mechanism for lazy-listing of frame axioms.

There are several formalisms for reasoning about actions. To our knowledge, none of them has been proved to have the interpolation property[3]. However, we shall use the extended propositional dynamic logic ($EPDL$), introduced by Zhang and Foo in [24], as the action description language and reasoning framework to take advantage of its sound and complete axiomatization and facilities for reasoning about action effects.

We remark that there have been several results about the the interpolation property of modal logic and dynamic logic but these results are not exactly what we want because a frame axiom may not be an arbitrary formula[16][17] [18].

2 Extended Propositional Dynamic Logic (EPDL)

In this section, we briefly summarize some basic facts of EPDL(see [24] for more details).

The language of $EPDL$ consists of a set $\mathbf{Flu}$ of fluent symbols and a set $\mathbf{Act}_P$ of primitive action symbols. There are two types of formulas: *propositional formula* $\varphi \in \mathbf{Fma_P}$, which does not include modal operators, and *ordinary formula* $A \in \mathbf{Fma}$. The syntax of an action $\alpha \in \mathbf{Act}$ is as same as programs in PDL.

The BNF rules of the language are as follows:

$\varphi ::= f \mid \neg\varphi \mid \varphi_1 \rightarrow \varphi_2$

[3] Some action logics are first-order or have some fragment which is first-order. This does not mean that such a logic has the interpolation property.

$A ::= f \mid \neg A \mid A_1 \to A_2 \mid [\alpha]A \mid [\varphi]A$
$\alpha ::= a \mid \alpha_1; \alpha_2 \mid \alpha_1 \cup \alpha_2 \mid \alpha^* \mid A?$
where $f \in \mathbf{Flu}$ and $a \in \mathbf{Act}_P$.

$[\gamma]A$ reads as "γ (always) causes A", where γ can be an action or a propositional formula. For instance, $[Shoot]\neg alive$ represent the causal relation "Shooting at a turkey kills the turkey"(See Example 1). $[\neg alive]\neg walking$ means "The death of the turkey causes it to be disable to walk". The dual operator $\langle\alpha\rangle$ of $[\alpha]$, defined as usual, reads as "α is executable and possibly causes A to be true". Specially, $\langle\alpha\rangle\top$ represents α is executable.

The axiomatic system of $EPDL$ consists of the following axiom schemes and inference rules:

(1). Axiom schemes:
• all tautologies of the propositional calculus.
• all axioms for compound programs (see [10]).
• EK axiom: $[\gamma](A \to B) \to ([\gamma]A \to [\gamma]B)$
• CW axiom: $[\varphi]A \to [\varphi?]A$
(2). Inference rules:
• MP: From A and $A \to B$, infer B.
• EN: From A, infer $[\gamma]A$.
• LE: From $\varphi_1 \leftrightarrow \varphi_2$, infer $[\varphi_2]A \leftrightarrow [\varphi_1]A$.
where φ, $\varphi_1, \varphi_2 \in \mathbf{Fma}_P$, $A \in \mathbf{Fma}$, $\alpha \in \mathbf{Act}$ and $\gamma \in \mathbf{Fma}_P \cup \mathbf{Act}$.

A formula A is a theorem of $EPDL$, denoted by $\vdash A$, if it can derived from the axiom system.

An *action description* of a dynamic system is a finite set of $EPDL$ formulas, which specifies the effects of actions in the system.

Let Σ be an action description. A formula A is a Σ*-theorem*, written by $\vdash^{\Sigma} A$, if it belongs to the least set of formulas which contains all the theorems of $EPDL$ and all the elements of Σ, and is closed under MP and EN.

If Γ is a set of formulas, then A is Σ*-provable* from Γ, written by $\Gamma \vdash^{\Sigma} A$, if there exists $A_1, \cdots, A_n \in \Gamma$ such that $\vdash^{\Sigma} (A_1 \wedge \cdots \wedge A_n) \to A$.

An action description Σ is *uniformly consistent* if $\nvdash^{\Sigma} \bot$. Γ is Σ*-consistent* if $\Gamma \nvdash^{\Sigma} \bot$.

Intuitively, $\Gamma \vdash^{\Sigma} A$ means A can be derived from the premisses Γ by the axioms of $EPDL$, action axioms in Σ and inference rules of $EPDL$.

Example 1 Consider the Yale Shooting Problem ([11]), which states that we have a turkey that gets killed if it is shot by a loaded gun. This problem can be described by the following action description:

$$\Sigma = \left\{ \begin{array}{l} \neg loaded \to [Load]loaded \\ loaded \to [Shoot]\neg alive \\ loaded \to [Shoot]\neg loaded \\ \langle Load\rangle\top, \langle Wait\rangle\top, \langle Shoot\rangle\top \end{array} \right\}$$

The first three sentences state the effects of action $Load$ and $Shoot$ on fluent $loaded$ and $alive$ (effect axioms). The last three represent the executability of actions (qualification axioms). Then we can have the following assertion:

$\{\neg loaded\} \vdash^{\Sigma} [Load; Shoot]\neg alive$.

A standard model M of $EPDL$ is a Σ-model if for any $B \in \Sigma$, $M \models B$. It has been proved that A is Σ-provable if and only if it is valid in all Σ-models. An action description Σ is uniformly consistent if and only if there exists a Σ-model.

3 Lazy-formalization

To model a dynamic system, effect axioms (which specify what is affected by actions) are generally listed in the action description unless effects of some actions are unknown. Often this is easy because most actions affect only few relevant facts. However listing all the frame axioms is tedious. They are much more numerous than effect axioms. In Example 1, only effect axioms were listed. There are nine frame axioms, such as $alive \rightarrow [Load]alive$, $loaded \rightarrow [Wait]loaded$ and etc., were not added. Without these axioms, the action description is not complete. We cannot even establish the intuitive assertion $\{\neg loaded, alive\} \vdash^{\Sigma} [Load]alive$.

In this section, we introduce a formal approach for reasoning about effects of actions with the frame axioms which are not included in action descriptions.

3.1 Supplementary frame axioms

We call a formula in the form $\varphi \wedge L \rightarrow [a]L$ a *frame axiom*, where φ is a propositional formula, a a primitive action and L is a literal. φ and L are called the *pre-condition* and the *body* of the axiom, respectively.

Definition 1 Let Γ be Σ-consistent. For a given formula A, if a set Δ of frame axioms satisfies that Γ is $\Sigma \cup \Delta$-consistent and $\Gamma \vdash^{\Sigma \cup \Delta} A$, then A is called to be *Σ-provable from Γ with Δ*, denoted by $\Gamma \vdash^{\Sigma}_{\Delta} A$. The elements of Δ are called *supplementary axioms* [4].

Roughly speaking, $\Gamma \vdash^{\Sigma}_{\Delta} A$ means that A can be derived from Γ under the action description Σ with the additional assumption Δ of frame axioms.

We generally call "$\Gamma \vdash^{\Sigma} A$" a *query*. If $\Gamma \vdash^{\Sigma}_{\Delta} A$, then Δ is called the frame axioms involved in the query.

Consider Example 1 again. We can prove that $\{\neg loaded\} \vdash^{\Sigma}_{\Delta} [Load; Wait; Shoot]\neg alive$, where $\Delta = \{loaded \rightarrow [Wait]loaded\}$.

3.2 Localization of supplementary frame axioms

We now investigate how lazy-formalization can be used to reason about effects of actions without an explicitly provided complete set of frame axioms. *Given an action description Σ of a system and a certain query "$\Gamma \vdash^{\Sigma} A$" about the system, which frame axioms would be really needed to answer the query?* Do they only depend on Σ, Γ and A, or even only depend on the query itself. Let us initially check the first possibility, which seems more intuitive. To make this more precise, we make the following:

Conjecture 1 *If $\Gamma \vdash^{\Sigma}_{\Delta} A$, then there exists a set Δ' of frame axioms such that*

1. $\Gamma \vdash^{\Sigma}_{\Delta'} A$,

2. $\Delta \vdash \Delta'$ and

3. every non-logical symbol which occurs in Δ' occurs in $\Sigma \cup \Gamma \cup \{A\}$.

[4] Note that some frame axioms could have been included in Σ.

This means that in order to answer the query "$\Gamma \vdash^{\Sigma} A$", the supplementary axioms really needed are the ones which are relevant to Σ, Γ and A.

We easily associate the conjecture with the Craig Interpolation Theorem (Criag IT) of first order logic([3]). Informally, we can transform $\Gamma \vdash^{\Sigma}_{\Delta} A$ into the inference relation:

$$\Gamma \cup \{[\mathbf{any}]B : B \in \Sigma\} \cup \{[\mathbf{any}]B : B \in \Delta\} \vdash A \tag{1}$$

Then we move all the formulas in Γ and $\{[\mathbf{any}]B : B \in \Sigma\}$ from the left hand side to the right hand side of the inference relation. The resulting expression will have the following form:

$$F_1(\Delta) \vdash F_2(\Gamma, \Sigma, A).$$

If $EPDL$ has Craig IT-like property, there exists an interpolant formula F such that $F_1(\Delta) \vdash F$, $F \vdash F_2(\Gamma, \Sigma, A)$ and all the non-logical symbols which occur in F occur in $F_1(\Delta)$ and $F_2(\Gamma, \Sigma, A)$. Granted that it has, we still can not affirm the conjecture because we can't guarantee that the formula F can be transformed into a set of frame axioms as it is required in the conjecture. In the other words, the conjecture requires a special form of interpolant, *an interpolant of frame axioms*. The following lemmas manifest partial feature of such an interpolant.

Lemma 1 *Let Γ be finite*[5]*. If $\Gamma \vdash^{\Sigma}_{\Delta} A$, then there exists a subset Δ' of Δ such that $\Gamma \vdash^{\Sigma}_{\Delta'} A$ and the body of each frame axiom in Δ' occurs in $\Sigma \cup \Gamma \cup \{A\}$.*

Lemma 2 *Let Γ be finite. If $\Gamma \vdash^{\Sigma}_{\Delta} A$, then there exists a subset Δ' of Δ such that $\Gamma \vdash^{\Sigma}_{\Delta'} A$ and the action symbol in each frame axiom in Δ' occurs in $\Sigma \cup \Gamma \cup \{A\}$.*

The results show that we can delete from Δ all the frame axioms in which either their bodies or action symbols do not occur in $\Sigma \cup \Gamma \cup \{A\}$. Now we might try deleting or substituting all the symbols contained in the pre-conditions of the frame axioms but which do not occur in $\Sigma \cup \Gamma \cup \{A\}$. Unfortunately, this is not always feasible:

Example 2 Let $\Sigma = \{\langle a \rangle \top\}$. We can verify that

$$\{f_1, f_2, [a](\neg f_1 \vee \neg f_2)\} \vdash^{\Sigma}_{\Delta} ([a]f_1 \vee [a]f_2) \tag{2}$$

where $\Delta = \{f_3 \wedge f_1 \rightarrow [a]f_1, \neg f_3 \wedge f_2 \rightarrow [a]f_2\}$. However, there is no set Δ' of frame axioms which satisfies the conditions in Conjecture 1.$\Box$

It is easy to see that the supplementary axiom we really need to answer the query (2) is $(f_1 \rightarrow [a]f_1) \vee (f_2 \rightarrow [a]f_2)$. We call such a disjunction of frame axioms *disjunctive frame axiom*. The example shows that disjunctive frame axioms can be introduced by using "alien" fluent symbols (f_3).

Conjecture 1 is therefore false. However, Example 2 is actually an illustration of the worst case. The following interpolation theorem declare that if we admit disjunctive frame axioms as supplementary axiom, the interpolant stated by Conjecture 1 exists.

Let Δ be a set of frame axiom. We call the set $\{B_1 \vee \ldots \vee B_n : B_1, \ldots, B_n \in \Delta\}$ the *disjunctive extension* of Δ.

Theorem 1 *Let Γ be finite. If $\Gamma \vdash^{\Sigma}_{\Delta} A$, then there exists a set Δ' of frame axioms such that*

[5] The finitary restriction is required by the local completeness of EPDL

1. $\Gamma \vdash^{\Sigma}_{\Psi} A$*, where* Ψ *is the disjunctive extension of* Δ'*,*
2. $\Delta \vdash \Delta'$*, and*
3. *every non-logical symbol in* Δ' *occurs in* $\Sigma \cup \Gamma \cup \{A\}$*.*

Conversely, if $\Gamma \vdash^{\Sigma}_{\Psi} A$*,* Ψ *is a set of disjunctive frame axioms, then there exists a set* Δ *of frame axioms such that* $\Gamma \vdash^{\Sigma}_{\Delta} A$*.*

We have obtained the soundness of lazy-formalization approach to the frame problem at the price of allowing the use of disjunctive frame axioms. We can also keep the original form of Conjecture 1 by prohibiting any potential use of disjunctive frame axioms at the price of the loss of reasoning about nondeterministic effects of actions; Pednault's Completeness Assumption and Reiter's Causal Completeness Assumption(see [19][22] are examples of such a restriction. For instance, in the Reiter's approach ([22]), the successor state axioms of Example 2 are

$f_1 \leftrightarrow [a]f_1$, $f_2 \leftrightarrow [a]f_2$.

The query (1) then become illegal because the premises are inconsistent with the action theory. In other words, Reiter's approach does not allow non-deterministic use of frame axioms.

4 Lazy-formalization with normal action descriptions

Theorem 1 shows that the lazy-formalization approach to the frame problem is reliable, and feasible if local effects are "small". Therefore, if we believe that we shall never have a query such as "Does a cough of an ET causes the light to be off?", we will never need to include the frame axiom: "A cough of an ET does not cause the light to be off" in our inference even if ET is a formal object in our language. The frame axioms involve the elements other than sw_1 and $light$ are also not required if they are not involved in any queries.

One might think that this solution is still unsatisfactory because we can only reduce the frame axioms to the language of $\Sigma \cup \Gamma \cup \{A\}$ rather than a query itself. If Σ is the specification of a huge system, it could involve a quite large set of symbols. Additionally, the frame axioms generated by the monotonic approach to the frame problem are generally expressible by the language of Σ (the set of effect axioms). So their frame axioms are at worst the same in size as that which lazy-formalization generates. What then is the advantage of lazy-formalization? The following section shows that lazy-formalization could do much better than the monotonic approach does. The reason is that most monotonic solutions exploit some syntactic restrictions, and with the same restrictions lazy-formalization can dramatically reduce the size of frame axioms further.

4.1 Normal action descriptions

In [25], a normalized form of action description was introduced to discuss consistency of action descriptions. An action description Σ is *normal* if each formula in Σ is in the form:

$$\varphi \to [a]L, \ \varphi \to \langle a \rangle \top,$$

called an *action law*, where $\varphi \in \mathbf{Fma}_P$, $a \in \mathbf{Act}_P$ and L is a fluent literal[6].

For any fluent f and any primitive action a, if we merge the action laws about f and a in the same form together, there will be at most three action laws about f and a in Σ as follows:

[6] To serve the purpose of the paper, we only consider deterministic action laws.

$$\varphi_0 \to \langle a \rangle \top, \varphi_1 \to [a]f, \varphi_2 \to [a]\neg f$$

We then call a normal action description Σ *safe* if for any a and f, the related action laws in Σ satisfy $\vdash \neg\varphi_0 \vee \neg\varphi_1 \vee \neg\varphi_2$. [7]

It is easy to see that the condition of safety is introduced to guarantee the consistency of action description.

Proposition 1 *[25] Let Σ be normal and safe, Γ be a set of propositional formulas. Γ is Σ-consistent if and only if Γ is consistent.*

This proposition make the Σ-consistency checking easier, and Definition 1 more accessible.

We remark that the normal form is quite expressive. Most normal forms in other action theories can be transformed into normal form (propositional case only). For instance, action descriptions in the *propositional* situation calculus language (i.e., there are no sorts *object* and function symbols in the language [22]) can be translated into normal form If we extend the normal form to allow an expression of causal laws and non-deterministic action laws [25], most components of action languages [7] can also be expressed by normal form. The same translation procedure will work for action descriptions in STRIPS [5].

The next interpolation theorem further reduces the search space for supplementary frame axioms.

Theorem 2 *Let Σ be normal and Γ be finite. If $\Gamma \vdash^{\Sigma}_{\Delta} A$ and $\Sigma \cup \Delta$ is safe, then there exists a subset Δ' of Δ such that $\Gamma \vdash^{\Sigma}_{\Delta'} A$ and each action symbol in Δ' occurs in $\Gamma \cup \{A\}$.*

The theorem shows that under the restriction of normality, the required frame axioms depend on only the query itself. This significantly reduced the number of frame axioms. For instance, to answer the query:

$\{\neg loaded, alive\} \vdash^{\Sigma} [Wait; Shoot]alive,$

we only need to consider the frame axioms about action $Wait$ and $Shoot$ (6 out of 9).

5 Discussion and conclusion

We introduced an inference mechanism, called *lazy-formalization*, for reasoning with incomplete action descriptions by postponing the listing of frame axioms until they are needed for answering a query. As a contribution to the meta-theory of reasoning about action, we presented two interpolation properties of action logic, based on dynamic logic, to show that the lazy formalization can mitigate and even avoid the frame problem in the following sense:

1. The frame axioms required for answering a query are the ones in which the non-logical symbols are in the action description and the query, provided we are allowed to use disjunctive frame axioms.
2. If the action description is written in normal form and is safe, the frame axioms can be further reduced to the ones in which the action symbols are in the query.

As we have mentioned in the introduction, all the existing solutions to the frame problem rest on some meta-hypotheses. The first result releases the lazy-formalization

[7] If there is no a relative form of the action law, say $\varphi_1 \to [a]f$, we assume $\varphi_1 \leftrightarrow \bot$.

approach from meta level assumption. Since most such kind of meta-hypotheses are essentially equivalent[6] and express a similar idea: effect of actions should be minimalised and action description should be localized, this result may be used in other formalisms of action to fill the gap between underlying logics and meta-hypotheses.

The second result provides a computational perspective to the lazy-formalization approach. If we express an action description in normal form as most formalisms of action do[6], the process for generating frame axioms is in linear time with respect to the size of effect axioms, which is exactly the same as Reiter's procedure. However, the inference for answering the query is normally in exponential time. Therefore, even a minor reduction of inputs would gain a significant improvement in computation. Therefore lazy-formalization can be viewed as a refinement of Reiter's solution from computational point of view [8]

We remark that although the results in the work are based on the propositional dynamic logic, the approach of lazy-formalization to the frame problem is applicable to other formalisms of action because the persistence of fluents is independent to its representation.

Appendix: Proofs of Theorems

Proof of Lemma 1: We shall remove from Δ the frame axiom which body does not occur in $\Sigma \cup \Gamma \cup \{A\}$ one by one.

Firstly we assume that we have only one supplementary frame axiom. Let $\Gamma \vdash^{\Sigma}_{\Delta} A$ and $\Delta = \{\varphi \wedge L \rightarrow [a]L\}$, in which the fluent symbol, say f_0, of L does not occur in $\Sigma \cup \Gamma \cup \{A\}$. We prove that $\Gamma \vdash^{\Sigma} A$.

By completeness of EPDL, $\Gamma \not\vdash^{\Sigma} A$ implies $\Gamma \not\models^{\Sigma} A$, that is, there exists a Σ-model $M = (W, \mathcal{R}, V)$ of language $\mathcal{L} \backslash \{f_0\}$ such that $M \models \Gamma$ but $M \not\models A$. We construct a model $M' = (W', \mathcal{R}', V')$ of language $\mathcal{L}$ as follows:

$W' = \{0, 1\} \times W$

For any $\alpha \in \mathbf{Act_P}$, $((i, u), (j, v)) \in R'_\alpha$ iff $i = j$ and $(u, v) \in R_\alpha$;

For any $\varphi \in \mathbf{Fma_P}$, $((i, u), (j, v)) \in R'_\varphi$ iff $i = j$ and $(u, v) \in R_\varphi$;

For any $f \in \mathbf{Flu}$, if $f \neq f_0$, $V(f) = \{(0, w) : w \in V(f)\} \cup \{(1, w) : w \in V(f)\}$; otherwise, $V(f) = \{0\} \times W$, that is, $V(f_0) = \{0\} \times W$.

Then we can make the following claims:

Claim 1: For any $B \in \mathbf{Fma}$, if B does not contain f_0, then $M' \models_{(i,w)} B$ if and only if $M \models_w B$

Claim 2: $M' \models \Gamma$ and $M' \not\models A$

Claim 3: M' is still a Σ-model.

Claim 4: M' is a $\Sigma \cup \Delta$-model.

The claim 1 can be easily verified by induction on the structure of formulas. The claim 2 and 3 is followed by the claim 1. For the last one, suppose that $M' \models_{(i,u)} \varphi \wedge L$. Then for any $(j, v) \in W'$ such that $((i, u), (j, v)) \in R'_a$, we have $i = j$ according to the construction of model M'. Hence $M' \models_{(j,v)} L$ no matter whether L is f_0 or $\neg f_0$. Therefore $M' \models \Delta$.

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[8] We may argue that lazy-formalization does not solve the frame problem because, in the worst case, the number of frame axioms is still as many as $O(\|actions\| \times \|fluents\|)$ whereas, in Reiter's solution, the number can be reduced to $O(\|fluents\|)$ by quantifying action variables. However, the first figure is counted in a local query language while the second is based on the global language. Moreover, the compact representation of action description with free action variables does not necessarily benefit computation. The instantiation of free variables is generally a necessary step in automatic reasoning.

According to the soundness of Σ-provability, $\Gamma \nvdash^{\Sigma\cup\Delta} A$, which contradicts $\Gamma \vdash^{\Sigma}_{\Delta} A$. Thus $\Gamma \vdash^{\Sigma} A$.

By following this step, we can remove all the supplementary frame axioms which bodies do not occur in $\Sigma \cup \Gamma \cup \{A\}$ one by one. □

Proof of Lemma 2: Without lost of generality, we suppose that Δ only contains one frame axiom $\varphi \wedge L \rightarrow [a]L$, where a does not occur in $\Sigma \cup \Gamma \cup \{A\}$. We prove that $\Gamma \vdash^{\Sigma}_{\Delta} A$ implies $\Gamma \vdash^{\Sigma} A$.

Suppose that $\Gamma \vdash^{\Sigma}_{\Delta} A$, we prove $\Gamma \vdash^{\Sigma} A$ by contraposition. So assume that $\Gamma \nvdash^{\Sigma} A$. Then there exists a Σ-model $M = (W, \mathcal{R}, V)$ of the language $\mathcal{L}\backslash\{a\}$ such that $M \models \Gamma$ but $M \not\models A$. We extend M to a model $M' = (W, \mathcal{R}, V)$ of language $\mathcal{L}$ as follows:

$W' = W$ and $V' = V$

$\mathcal{R}'$ is as same as R except for the accessibility relation R'_a for action a and each R'_α for the compound action α involving a. We define R'_a in the following and the others can be done by standard model conditions.

For any $u', v' \in W'$, $(u', v') \in R_a$ iff $u' = v'$ and $M' \models_u \varphi$. Obviously $M' \models \Delta$. Hence we have that M' is a $\Sigma \cup \Delta$-model, $M' \models \Gamma$ and $M' \not\models A$, which contradicts $\Gamma \vdash^{\Sigma}_{\Delta} A$. □

Proof of Theorem 1

According to the lemma 1 and 2, we can delete from Δ all the frame axioms in which either their bodies or action symbols do not occur in $\Sigma \cup \Gamma \cup \{A\}$. So we just need to consider the fluent symbols which occur in the pre-conditions of frame axioms in Δ but not in $\Sigma \cup \Gamma \cup \{A\}$.

For convenience, we write a frame axiom $\varphi \wedge L \rightarrow [a]L$ in the form $\varphi \rightarrow \gamma$, where $\gamma = \neg L \vee [a]L$. Then all the elements of Δ can be listed uniformly as follows:

$\varphi_1 \rightarrow \gamma_1, \cdots, \varphi_n \rightarrow \gamma_n$

Suppose that f_0 is a fluent symbol which occurs in Δ but does not occur in $\Sigma \cup \Gamma \cup \{A\}$. According to the above assumption, f_0 can only occur in some pre-conditions φ_i. For each such a pre-condition φ_i, let ψ'_i be the result of everywhere replacing occurrences of f_0 in φ_i by $\top$, and ψ''_i by $\bot$.

Let $\psi = \bigwedge_{i=1}^{n} (\neg\psi'_i \vee \gamma_i) \vee \bigwedge_{i=1}^{n} (\neg\psi''_i \vee \gamma_i)$

We prove that $\Gamma \vdash^{\Sigma}_{\Delta} A$ implies $\Gamma \vdash^{\Sigma\cup\{\psi\}} A$. Firstly, it is not hard to verify the following facts:

Fact 1: $\varphi_i \dashv\vdash (f_0 \wedge \psi'_i) \vee (\neg f_0 \wedge \psi''_i)$.

Fact 2: $\Delta \vdash \psi$.

Fact 3: $\Sigma \cup \{\psi\}$ is uniformly consistent.

Suppose that $\Gamma \vdash^{\Sigma\cup\{\psi\}} A$ is not true. Then there must be a $\Sigma \cup \{\psi\}$-model $M = (W, \mathcal{R}, V)$ such that $M \models \Gamma$ but for some $w_0 \in W$, $M \models_{w_0} \neg A$. Now we transform M into a $\Sigma \cup \Delta$-model $M' = (W, \mathcal{R}, V')$ by changing valuation of fluent f_0 as follows: for any $w \in W$,

1. if $M \models_w \bigwedge_{i=1}^{n} (\neg\psi'_i \vee \gamma_i)$, then let $w \in V'(f_0)$; otherwise $w \notin V'(f_0)$. In this case, $M \models_w \bigwedge_{i=1}^{n} (\neg\psi''_i \vee \gamma_i)$ because $M \models_w \psi$.

2. $V'(f) = V(f)$ if $f \neq f_0$.

Then we prove M' is a $\Sigma \cup \Delta$-model. In fact, M' is a Σ-model and for any $w \in W$, if $w \in V'(f_0)$, that is, $M' \models_w \bigwedge_{i=1}^{n} (\neg\psi'_i \vee \gamma_i)$, then $M' \models_w (\neg f_0 \vee \bigwedge_{i=1}^{n} (\neg\psi'_i \vee \gamma_i)) \wedge (f_0 \vee \bigwedge_{i=1}^{n} (\neg\psi''_i \vee \gamma_i))$. It follows $M' \models_w \bigwedge_{i=1}^{n} (\varphi_i \rightarrow \gamma_i)$; If $w \notin V'(f_0)$, $M' \models_w \bigwedge_{i=1}^{n} (\neg\psi''_i \vee \gamma_i)$, so $M' \models_w \bigwedge_{i=1}^{n} (\varphi_i \rightarrow \gamma_i)$. That means $M' \models_w \Delta$. Therefore M' is a $\Sigma \cup \Delta$-model. Since f_0 does not occur in $\Gamma \cup \{A\}$, $M' \models \Gamma$ and $M' \models_w \neg A$. Thus $\Gamma \not\models^{\Sigma\cup\Delta} A$, which contradicts $\Gamma \vdash^{\Sigma}_{\Delta} A$. Therefore $\Gamma \vdash^{\Sigma}_{\Delta} A$ implies $\Gamma \vdash^{\Sigma\cup\{\psi\}} A$.

In this way, we can remove all the fluent symbols which occur in Δ but do not occur in $\Sigma \cup \Gamma \cup \{A\}$, and obtain a set Ψ of formulas such that $\Gamma \vdash^{\Sigma}_{\Delta} A$ implies $\Gamma \vdash^{\Sigma\cup\Psi} A$. Obviously, Ψ can be expressed by a set of disjunctive frame axioms.

For the reverse, assume that Ψ is a set of formulas, each of which has the form

$$\bigwedge_{i=1}^{m}(\neg\psi'_i \vee \gamma_i) \vee \bigwedge_{j=1}^{n}(\neg\psi''_j \vee \gamma_j) \tag{3}$$

where ψ'_i and ψ''_i are any propositional formulas, γ_i has the form of $\neg L \vee [a]L$, and all the fluent and action symbols occur in them occur in $\Sigma \cup \Gamma \cup \{A\}$. We need to prove that if $\Gamma \vdash^{\Sigma\cup\Psi} A$, then there exists is a set Δ of frame axioms such that $\Gamma \vdash^{\Sigma}_{\Delta} A$.

To construct such a Δ, we start from an empty set. For each formula in the form of equation (3) in Ψ, we introduce a new fluent symbols f, and add the following "axioms" into Δ :

$f \wedge \psi'_i \rightarrow \gamma_i$, $\neg f \wedge \psi''_j \rightarrow \gamma_j$, $(i = 1, \cdots, m$ and $j = 1, \cdots, n)$

Now we prove that if Γ is $\Sigma \cup \Psi$-consistent, then $\Gamma \vdash^{\Sigma\cup\Psi} A$ implies $\Gamma \vdash^{\Sigma}_{\Delta} A$.

To this end, we prove that Γ is $\Sigma \cup \Delta$-consistent. Since Γ is $\Sigma \cup \Psi$-consistent, there exists a $\Sigma \cup \Psi$-model $M = (W, \mathcal{R}, V)$ such that for some $w_0 \in W$, $M \models_{w_0} \Gamma$. Construct a model $M' = (W', \mathcal{R}', V')$ of the language with the new introduced fluent symbols. Let $W' = W$ and $\mathcal{R}' = \mathcal{R}$. V' is different from V only in the valuation of new fluent symbols:

For any $w \in W$, $w \in V'(f)$ iff $M' \models_w \bigwedge_{i=1}^{m}(\neg\psi'_i \vee \gamma_i)$

Thus, for each $f \wedge \psi'_i \rightarrow \gamma_i \in \Delta$, if $M' \models_w \neg f$, then $M' \models_w f \wedge \psi'_i \rightarrow \gamma_i$; if $M' \models_w f$, then $M' \models_w \bigwedge_{i=1}^{m}(\neg\psi'_i \vee \gamma_i)$, so $M' \models_w \neg\psi'_i \vee \gamma_i$. This means $M' \models_w f \wedge \psi'_i \rightarrow \gamma_i$.

For each $\neg f \wedge \psi''_j \rightarrow \gamma_j \in \Delta$, if $M' \models_w f$, then $M' \models_w \neg f \wedge \psi''_j \rightarrow \gamma_j$. If $M' \models_w \neg f$, then by $M \models_w \bigwedge_{i=1}^{m}(\neg\psi'_i \vee \gamma_i) \vee \bigwedge_{j=1}^{n}(\neg\psi''_j \vee \gamma_j)$, we have that $M' \models_w \bigwedge_{j=1}^{n}(\neg\psi''_j \vee \gamma_j)$, so $M' \models_w \neg\psi''_j \vee \gamma_j$. That is $M' \models_w \neg f \wedge \psi''_j \rightarrow \gamma_j$. It's easy to see that M' is a Σ-model and $M' \models_w \Gamma$. Therefore we have proved that M' is a $\Sigma \cup \Delta$-model and Γ is $\Sigma \cup \Delta$-consistent.

Because $\Delta \vdash \Psi$, $\Gamma \vdash^{\Sigma\cup\Psi} A$ implies $\Gamma \vdash^{\Sigma}_{\Delta} A$. □

Proof of Theorem 2

Without loss of generality, we suppose that Δ only contains the supplementary axioms on a primitive action symbol a which does not in $\Gamma \cup \{A\}$. We prove that $\Gamma \vdash^{\Sigma}_{\Delta} A$ implies $\Gamma \vdash^{\Sigma} A$.

Assume that $\Gamma \not\vdash^{\Sigma} A$. Then there exists a canonical model $M^C = (W^C, \mathcal{R}^C, V^C)$ of Σ such that

$W^C = \{w^C : w^C$ is a maximal Σ-consistent set of formulas$\}$

and $M^C \models \Gamma$ but $M^C \not\models A$. Let $M^\Gamma = (W^\Gamma, \mathcal{R}^\Gamma, V^\Gamma)$ be the $FL(\Sigma \cup \Delta \cup \Gamma \cup \{A\})$-filtration of M^C. It is easy to see that M^Γ is a Σ-model, $M^\Gamma \models \Gamma$ and $M^\Gamma \not\models A$. Now we transform M^Γ into a $\Sigma \cup \Delta$-model if it is not. Assume that $\Sigma \cup \Delta$ contains the following causal laws about a :

$\varphi_0 \rightarrow \langle a\rangle\top$, $\varphi_1 \rightarrow [a]L_1, \cdots, \varphi_m \rightarrow [a]L_m$

Let $M = (W, \mathcal{R}, V)$ be a model which is as same as M^Γ except for the accessibility relation R_a for a and relative accessibility relations for compound actions involving a. We define R_a as follows: [9]

for any $w, w' \in W$, $(w, w') \in R_a$ iff

[9] The other undefined accessibility relations for compound actions are ready to be obtained by standard model conditions.

1. $M \models_w \varphi_0$;

2. for each $i(1 \le i \le m)$, if $M \models_w \varphi_i$, then $M \models_{w'} L_i$;

It is easy to see that $M \models \Gamma$ and $M \not\models A$. It is not hard to prove that M is a $\Sigma \cup \Delta$-model. Therefore $\Gamma \not\vdash^{\Sigma \cup \Delta} A$, which contradicts $\vdash^{\Sigma}_{\Delta} A$. So we can conclude that $\vdash^{\Sigma}_{\Delta} A$ implies $\vdash^{\Sigma} A$. □

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