Default Reasoning and Belief Revision: A Syntax-Independent Approach

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Abstract As an important variant of Reiter's default logic, Poole (1988) developed a nonmonotonic reasoning framework in the classical first-order language. Brewka and Nebel extended Poole's approach in order to enable a representation of priorities between defaults. In this paper a general framework for default reasoning is presented, which can be viewed as a generalization of the three approaches above. It is proved that the syntax-independent default reasoning in this framework is identical to the general belief revision operation introduced by Zhang *et al.* (1997). This result provides a solution to the problem whether there is a correspondence between belief revision and default logic for the infinite case. As a by-product, an answer to the question, raised by Mankinson and Gärdenfors (1991), is also given about whether there is a counterpart contraction in nonmonotonic logic.

Keywords nonmonotonic logic, default reasoning, belief revision

1 Introduction

Reiter's default logic[1] is one of the most well-developed systems of nonmonotonic reasoning. A number of variants of default logic[2−4] have been developed in the past several years. Poole's approach[4] is simple and natural. In his system facts and defaults are all represented by the classical first-order language instead of adding some specific non-classical rules. There is a disadvantage of Poole's approach, however, that it does not allow to represent priorities between defaults. Brewka's *preferred sub-theories* developed in [2, 5] have efficiently overcome this shortage. Nebel[3] combined both approaches of Poole and Brewka, and developed a system for default reasoning, called *ranked default theory* (RDT). As a very important result, Nebel established a relation between RDT and revised belief revision (called *prioritized base revision*), which strengthened Makinson and Gärdenfors' result[6] about the connections between nonmonotonic reasoning and belief revision. The goal of this paper is to further extend Nebel's results so as to obtain more direct and general connections between default reasoning and belief revision. We will present a general framework for default reasoning, which can be viewed as a generalization of the three approaches above. We prove that the syntax-independent default reasoning in this framework is strictly identical to the general belief revision operation, introduced by Zhang *et al.*[7,8]. This result provides a solution to the problem whether there is a correspondence between belief revision and default logic for the infinite case. As a by-product, we also give an answer to the question, raised by Makinson and Gärdenfors[6], of whether there is a counterpart contraction in nonmonotonic logic.

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1.1 Preliminaries

Throughout this paper, we consider the first-order language L with the standard logical connectives $\neg, \vee, \wedge, \rightarrow$ and $\leftrightarrow$. A sentence in L is a well-formed formula without free variables. We denote individual sentences by A, B, or C, and denote sets of sentences by K, F, Γ etc. The notation $\vdash$ means classical first-order derivability and Cn the corresponding closure operator, i.e., $Cn(\Gamma) = \{A \in L | \Gamma \vdash A\}$.

1.2 Poole's System

Poole[4] described a system for default reasoning. In his framework it is assumed that the user provides two sets F and Δ. F is a set of closed formulae, which is treated as hard facts. Δ is a set of formulae, called the set of possible hypotheses.

A *default theory* is a pair (F, Δ). A *scenario* of (F, Δ) is a set $D \cup F$ where D is a set of ground instances of elements of Δ such that $D \cup F$ is consistent. g is *explainable* from (F, Δ) iff there is a scenario of F and Δ which implies g. An *extension* of (F, Δ) is the set of logical consequences of a maximal (with respect to set inclusion) scenario of (F, Δ).

In order to block unwanted transitivities and contrapositives of some defaults, Poole introduced a set C of closed formulae called the set of *constraints*. A default theory with constraints is then a triple (F, Δ, C) and the definition of scenario is revised as follows.

A *scenario* of (F, Δ, C) is a set $D \cup F$, where D is a set of ground instances of elements of Δ such that $D \cup F \cup C$ is consistent. The definitions of explainability and extension are correspondingly changed.

1.3 Brewka's Generalization

Poole's approach is simple and elegant, but it does not provide a way to express priorities between defaults. In [2, 5] Brewka presented a generalization of Poole's system which allows one to do this exactly. He introduced several layers of possible hypotheses representing different degrees of reliability instead of only two — facts and defaults — as in Poole's system.

A default theory in Brewka's framework is a tuple $T = (T_1, \ldots, T_n)$, where each T_i is a set of classical first-order formulae. A *preferred sub-theory* of T is a set $S = S_1 \cup \cdots \cup S_n$, where for all k $(1 \leq k \leq n)$ $S_1 \cup \cdots \cup S_k$ is a maximal consistent subset of $T_1 \cup \cdots \cup T_k$. The deductive closure $Cn(S)$ of S is called an *extension* of T. It is easy to see that Poole's system is a special case of Brewka's, where $n = 2$.

1.4 Nebel's Generalization

Nebel[3] combined both approaches of Poole's and Brewka's, and obtained a common generalization. A *ranked default theory* (RDT) is a pair $\Delta = (D, F)$, where D is a finite sequence $(D_1, \ldots, D_n)$ of finite sets of propositions treated as ranked defaults and F is a finite set of propositions interpreted as hard facts. An *extension* of Δ is a deductively closed set of propositions

$$E = Cn\Big(\Big(\bigcup_{i=1}^{n} R_i\Big) \cup F\Big)$$

such that for all i with $1 \leq i \leq n$:

1. $R_i \subseteq D_i$,
2. R_i is set-inclusion maximal among the subsets of D_i such that $(\bigcup_{j=1}^{i} R_j) \cup F$ is consistent.

It is not difficult to see that Nebel's RDT $\Delta = (\{D_1, \ldots, D_n\}, F)$ can be translated to Brewka's default theory $T = (F, D_1, \ldots, D_n)$ in the case of finite propositional logic.

1.5 Motivating Example

Both Brewka's and Nebel's generalizations provide orders for defaults. They share a common drawback. However, they do not satisfy the principle of irrelevance of syntax. The following example can illustrate this problem. For more discussions, we refer the reader to Nebel (1992) ([9], p.59).

Example 1. A stockjobber hesitated about whether to buy shares of company Alpha or company Beta. He consulted with four experts in stock market.

The first one said: "You should buy Alpha, even in large numbers."

The second one said: "You may buy Alpha, but not in large numbers. I suggest that you buy some of Beta".

The third one said "You can't buy Beta".

The forth one said "You can buy Beta".

Let A="buy Alpha", B="buy Beta", C="buy Alpha in large numbers", then the stockjobber received the following information:

$D_1 = \{A \wedge C\}$, $D_2 = \{A \wedge \neg C \wedge B\}$, $D_3 = \{\neg B\}$, $D_4 = \{B\}$.

Suppose that information in D_i is more believed by the stockjobber than that in D_j if $i < j$. Then in Nebel's system, $\Delta = (\{D_1, \ldots, D_4\}, \phi)$ has a unique extension: $E = Cn(\{A \wedge C, \neg B\})$.

If the set of information is represented as

$D'_1 = \{A, C\}$, $D'_2 = \{A, \neg C, B\}$, $D'_3 = \{\neg B\}$, and $D'_4 = \{B\}$,

then $E' = Cn(\{A, C, B\})$ is the unique extension of $\Delta' = (\{D'_1, \ldots, D'_4\}, \phi)$.
This could be argued to be reasonable.

2 Perfect-Ordered Partition

In order to describe degrees of reliability of information, Zhang in [7] introduced a notion of total-ordered partition, which has been used to establish the general belief revision theory. In this paper we will reinforce this structure to specify the intended framework of default reasoning.

Definition 2.1. *Let Γ be a set of sentences, $\mathcal{P}$ a partition of Γ, $<$ a well-ordering relation on $\mathcal{P}$. The triple $\Sigma = (\Gamma, \mathcal{P}, <)$ is called a well-ordered partition* (*WOP*) *of Γ.*

For a well-ordered partition $\Sigma = (\Gamma, \mathcal{P}, <)$, if the order-type of $\mathcal{P}$ is η, we write $\mathcal{P}$ as a sequence $\{\Gamma_\alpha : \alpha < \eta\}$. If $A \in \Gamma_\alpha$, α is called the *rank* of A, denoted by $b(A)$. The idea is that different ranks of sentences represent different degrees of reliability. The lower rank a sentence is, the higher degree of reliability it has. When $R \subseteq \Gamma$, the following notations are useful:

$$R_\alpha \stackrel{\text{Def}}{=} R \cap \Gamma_\alpha, \qquad R_{<\alpha} \stackrel{\text{Def}}{=} \bigcup_{\gamma<\alpha} R_\gamma, \qquad R_{\leq\alpha} \stackrel{\text{Def}}{=} \bigcup_{\gamma\leq\alpha} R_\gamma.$$

As in [9], we define $\Gamma \Downarrow F$, for any set F of sentences, as the family of all subsets, $\Delta = \bigcup_{\alpha<\eta} R_\alpha$, of Γ, where for any $\alpha < \eta$, R_α is a maximal subset of Γ_α such that $R_{\leq\alpha} \cup F$ is consistent.

Just like well-ordered partition is a strengthening of total-ordered partition, we introduce the following notion to strengthen the notion of nice-ordered partition.

Definition 2.2. *A well-ordered partition, $\Sigma = (\Gamma, \mathcal{P}, <)$, of Γ is called a perfect-ordered partition* (*POP*) *if it satisfies the following Logical Constraint*:

(*L*) *If* $A_1, \ldots, A_n \vdash B$, *then* $\sup\{b(A_1), \ldots, b(A_n)\} \geq b(B)$.

The following lemma shows that it is possible to lift up a POP from a WOP automatically if the original partition does not satisfy (L). Let $\Sigma = (\Gamma, \mathcal{P}, <)$ be a well-ordered partition of Γ, and η be the order-type of $\mathcal{P}$. Let $K = Cn(\Gamma)$. We define an ordinal η^K and a new partition $\mathcal{P}^K = \{K_\alpha : \alpha < \eta^K\}$ of K as follows:

Let $K_0 = \Gamma_0$ and if the sets K_ξ, $\xi < \alpha$ have been defined, but η^K has not been defined, then

1. if $\bigcup_{\xi<\alpha} K_\xi = K$, then $\eta^K = \alpha$;
2. otherwise, define $K_\alpha = (Cn(\Gamma_{\leq\gamma(\alpha)}))/K_{<\alpha}$, where $\gamma(\alpha)$ is the smallest ordinal γ such that $\Gamma_\gamma \cap K_{<\alpha}$ is non-empty.

Lemma 2.1. *If $\Sigma = (\Gamma, \mathcal{P}, <)$ is a well-ordered partition of Γ and $K = Cn(\Gamma)$, then $\Sigma^K = (K, \mathcal{P}^K, <^K)$ is a perfect-ordered partition of K. Moreover, if Σ is a POP of Γ, then for any $A \in \Gamma, b^\Gamma(A) = b^K(A)$.*

We will call Σ^K as the POP of K induced by Σ.

Proof. It is not difficult to prove by induction that $\mathcal{P}^K$ is a partition of K and is well-ordered. So we are left to show that $\mathcal{P}^K$ is nice. Let $b^K(A)$ be the rank of A with respect to Σ^K. For any sentences $A_1, \ldots, A_n$ and B in K, suppose that $A_1, \ldots, A_n \vdash B$, and $\alpha = \max\{b^K(A_1), \ldots, b^K(A_n)\}$. Then $P^K_{\leq\alpha} \vdash B$. It follows that $Cn(P_{\leq\gamma(\alpha)}) \vdash B$, i.e., $B \in Cn(P_{\leq\gamma(\alpha)})$. So we have $B \in P^K_\alpha$ or $B \in P^K_{<\alpha}$ by the construction of P^K_α. □

3 General Belief Revision

Much of the work in belief revision is based on the studies of Alchourrón, Gärdenfors and Markinson[10], who have developed a framework, which we call the AGM theory for analyzing this process. Zhang *et al.*[8] presented an extended system of AGM's theory which enables a treatment of belief revision by sets of sentences, especially infinite sets. The extended revision and contraction operators were called general ones. A set of postulates was introduced in [7, 8] to model the general revision operation. A function '$\otimes$' for belief set K and set of sentences F in L is called a *general revision function* if it satisfies the following nine postulates:

($\otimes$1) $K \otimes F = Cn(K \otimes F)$.
($\otimes$2) $F \subseteq K \otimes F$.
($\otimes$3) $K \otimes F \subseteq K + F$.
($\otimes$4) If $K \cup F$ is consistent, then $K + F \subseteq K \otimes F$.
($\otimes$5) $K \otimes F$ is inconsistent iff F is inconsistent.
($\otimes$6) If $Cn(F_1) = Cn(F_2)$, then $K \otimes F_1 = K \otimes F_2$.
($\otimes$7) $K \otimes (F_1 \cup F_2) \subseteq K \otimes F_1 + F_2$.
($\otimes$8) If $F_2 \cup (K \otimes F_1)$ is consistent, then $(K \otimes F_1) + F_2 \subseteq K \otimes (F_1 \cup F_2)$.
($\otimes LP$) $K \otimes F = (\bigcap_{\bar{F} \subseteq_f Cn(F)} K \otimes \overline{F}) + F$.

Here $F \subseteq_f \Gamma$ denotes that F is a finite subset of Γ.

The corresponding contraction operator, called the general contraction, can be directly defined as follows:

(Def $\ominus$) $\quad K \ominus F = (K \otimes F) \cap K$

Note that there is an important difference between the general contraction and AGM's[10]. In AGM's system, contracting K with sentence A means removing A and some sentences from K so that the resulting belief set fails to imply A. In the general case, contracting K with F means removing some of sentences of K so that the remained set is closed and consistent with F.

In [8], the author employed the notion of nice-ordering partition to construct the general revision and contraction operators. These constructions can be simplified if the partition of belief set is perfect.

Definition 3.1. *Let $\Sigma = (K, \mathcal{P}, <)$ be a POP of K, and F a set of sentences. Define $K \otimes F$, called POP revision of K by F, as follows:*

$$K \otimes F = (\bigcap K \Downarrow F) + F$$

According to (Def $\ominus$), the corresponding contraction operation $\ominus$ can be constructed in the following way:

i) if F is inconsistent, $K \ominus F = K$;

ii) if F is consistent,

$$K \ominus F = (\bigcap K \Downarrow F)$$

Theorem 3.1.[8] *The function* $\otimes$ *satisfies* $(\otimes 1) - (\otimes 8)$ *and* $(\otimes LP)$.

An interesting observation in this context is that we can use the structure of well-ordered partition to extend Nebel's belief base revision[9].

Definition 3.2. *Let* C, F *be any sets of sentences where* C *is interpreted as a belief base.* Σ *is a well-ordered partition of* C. *We define* $C\hat{\otimes}F$, *called prioritized base revision w.r.t.* Σ, *as follows:*

$$C\hat{\otimes}F = (\bigcap_{\Gamma \in C \Downarrow F} Cn(\Gamma)) + F$$

We will use this notion to strengthen some results of Nebel's.

4 Belief Revision and Default Reasoning

This section will give three types of generalizations of Nebel's RDT and discuss their relationship with the general belief revision.

In the following two subsections, by a default theory, we mean a pair $\Delta = (D, F)$, where D and F are both sets of sentences in L, interpreted as "defaults" and "facts", respectively.

Definition 4.1. *Let* $\Delta = (D, F)$ *be a default theory. If there is a well-ordered partition* $\Sigma = (D, \mathcal{P}, <)$ *of* D, *we call* Δ *as a well-ordered partitioned default theory* (*WOP DT*).

4.1 A Direct Generalization of Nebel's System: Syntax-Dependent Approach

We first present a syntax-based approach to WOP DT which can be viewed as the most direct generalization of Nebel's approach.

Definition 4.2. *Let* $\Delta = (D, F)$ *be a WOP DT with a WOP* $\Sigma = (D, \mathcal{P}, <)$. *A deductively closed set* E *of sentences is called a syntax-dependent extension of* Δ *if there is a* $\Gamma \in D \Downarrow F$ *such that* $E = Cn(\Gamma \cup F)$[1].

A sentence A in L is strong provable in Δ, denoted by $\Delta \mid\!\sim A$, if for all extensions E of Δ, $A \in E$.

Nebel[9] gave a relationship between reasoning in RDT and prioritized base revision. He provided a method to translate reasoning in RDT into belief revision in the case of finite propositional logic. This result can be generalized to WOP DT for the case that sets of facts are finite modulo logical equivalence.

Proposition 4.1. *Let* $\Delta = (D, F)$ *be a WOP DT with a WOP* $\Sigma = (D, \mathcal{P}, <)$. *If* F *is finite modulo logical equivalence, then for all* $A \in L$:

$$\Delta \mid\!\sim A \text{ iff } A \in D\hat{\otimes}F$$

Proof. By comparing the definition of extension with the construction of prioritized base revision, it is not difficult to know that we only need to show

$$\bigcap_{\Gamma \in D \Downarrow F} Cn(\Gamma \cup F) = Cn\Big(\Big(\bigcap_{\Gamma \in D \Downarrow F} Cn(\Gamma)\Big) \cup F\Big)$$

"$\supseteq$" is obvious. To prove the other inclusion, consider that $A \in \cap_{\Gamma \in D \Downarrow F} Cn(\Gamma \cup F)$. Then, for every $\Gamma \in D \Downarrow F$, $\Gamma \cup F \vdash A$. Since F is finite modulo logical equivalence, there exists a sentence B such that $Cn(B) = Cn(F)$. So $\Gamma \vdash \neg B \vee A$, i.e., $\neg B \vee A \in Cn(\Gamma)$. This means $\neg B \vee A \in \cap_{\Gamma \in D \Downarrow F} Cn(\Gamma)$, hence, $A \in Cn((\cap_{\Gamma \in D \Downarrow F} C_n(\Gamma)) \cup F)$. □

[1] It is easy to see that this definition is identical to Nebel's when $\mathcal{P}$ and F are finite.

4.2 A General Framework for Default Reasoning: Syntax-Independent Approach

It is easy to see that Poole's system without constraints is a limiting case of syntax-dependent WOP DT when $\mathcal{P} = \{\mathcal{D}\}$ and Nebel's RDT is the special case when η_D is finite. Unfortunately, as pointed out by Nebel in [9], the inference relation $\Delta \mid\!\sim$ generated by syntax-dependent extensions fails to satisfy the rational monotony and syntax irrelevance. Now we modify the above definition into the following form.

Definition 4.3. *Let $\Delta = (D, F)$ be a WOP DT with a WOP $\Sigma = (D, \mathcal{P}, <)$, and η be the order-type of $\mathcal{P}$.*

A set of sentences E is an extension of Δ if $E = Cn((\cup_{\alpha<\eta} R_\alpha) \cup F)$ and, for all $\alpha < \eta$, $R_\alpha \subseteq Cn(D_{\leq\alpha})$ and R_α is maximal (with respect to set-inclusion) among the subsets of $Cn(D_{\leq\alpha})$ such that $R_{\leq\alpha} \cup F$ is consistent.

A sentence A is strong provable in Δ, denoted by $\Delta \mid\!\sim A$, iff for all extensions E of Δ, $A \in E$.

It is not difficult to show that extensions of a default theory defined above are irrelevant to its representation. So we will call this kind of approach for default reasoning as syntax-independent[2]. More precisely, we have:

Proposition 4.2. *For $i = 1, 2$, let $\Delta^i = (D^i, F^i)$ be a WOP DT with a WOP $\Sigma^i = (D^i, \mathcal{P}^i)$ and η^i be the order-type of $\mathcal{P}^i$. If $\eta^1 = \eta^2 = \eta$, $Cn(F^1) = Cn(F^2)$ and for all $\alpha < \eta$, $Cn(P^1_\alpha) = Cn(P^2_\alpha)$, then*

E is an extension of Δ^1 iff it is an extension of Δ^2.

Lemma 4.1. *Let K be a closed set of sentences. For any set,*

$$\bigcap_{\Gamma \in K \Downarrow F} (\Gamma + F) = \Big(\bigcap_{\Gamma \in K \Downarrow F} \Gamma \Big) + F$$

Proof. The direction '$\supseteq$' is obvious. For the other direction, suppose that $K \cup F$ is inconsistent (otherwise, the desired conclusion holds trivially). Then there exists a finite subset $\bar{F}$ of F such that $\neg(\wedge\bar{F}) \in K$. For any $A \in \bigcap_{\Gamma \in K \Downarrow F} (\Gamma + F)$, if $A \notin \bigcap_{\Gamma \in K \Downarrow F} \Gamma + F$, $\neg(\wedge\bar{F}) \vee A \notin \bigcap_{\Gamma \in K \Downarrow F} \Gamma$, i.e., there is $\Gamma' \in K \Downarrow F$ such that $\neg(\wedge\bar{F}) \vee A \notin \Gamma'$. Hence $\Gamma' \cup \{\neg(\wedge\bar{F}) \vee A\} \cup F$ is inconsistence, i.e, $\Gamma' \cup F \vdash \wedge\bar{F} \wedge \neg A$, or, $\Gamma' \cup F \vdash \neg A$. On the other hand, from $A \in \bigcap_{\Gamma \in K \Downarrow F} (\Gamma + F)$, we have $A \in \Gamma' + F$, so $\Gamma' \cup F \vdash A$, which is in contradiction with the consistency of $\Gamma' \cup F$. □

The following theorem shows that the syntax-independent default reasoning is strictly correspondent to belief revision operation.

Theorem 4.1. *Let $\Delta = (D, F)$ be a WOP DT with a WOP $\Sigma^D = (D, \mathcal{P}^D, <^D)$ of D. Let $K = Cn(D)$ and $\Sigma^k = (K, \mathcal{P}^K, <^K)$ be the POP induced by Σ^D. Then for all sentences A in L,*

$$\Delta \mid\!\sim A \text{ iff } A \in K \otimes F$$

Proof. By Definition 3.1 and Lemma 4.1 we have:

$$K \otimes F = (\cap K \Downarrow F) + F = \bigcap_{\Gamma \in K \Downarrow F} (\Gamma + F) = \bigcap_{\Gamma \in K \Downarrow F} Cn(\Gamma \cup F)$$

Thus suffice it to prove the following condition:

$$E \text{ is an extension of } \Delta \text{ iff } E = Cn(\Gamma \cup F), \text{ where } \Gamma \in K \Downarrow F.$$

[2] Related researches can be seen in [11], which are based on model-theoretic approach.

Let η, η' be the order-types of $\mathcal{P}^D, \mathcal{P}^K$, respectively.

"$\Rightarrow$": Assume that E is an extension of Δ. By the definition of extensions, there exists a sequence $\{R_\alpha : \alpha < \eta\}$ such that $E = Cn(\cup_{\alpha<\eta} R_\alpha \cup F)$, where for all $\alpha < \eta$, R_α is a maximal subset of $Cn(D_{\leq\alpha})$ such that $R_{\leq\alpha} \cup F$ is consistent.

For all $\alpha < \eta'$, let $R'_\alpha = R_{\leq\gamma(\alpha)} \cap K_\alpha$ where $\gamma(\alpha)$ is the smallest ordinal γ such that $D_\gamma \cap K_{<\alpha}$ is non-empty. Set that $\Gamma \equiv \cup_{\alpha<\eta'} R'_\alpha$. It remains only to show that $\Gamma \in K \Downarrow F$. To this end, suffice it to verify that for all $\alpha < \eta'$, R'_α is maximal among the subsets of K_α such that $R'_{\leq\alpha} \cup F$ is consistent. Assume that $A \in K_\alpha$ and $A \notin R'_\alpha$. Then $A \in Cn(D_{\leq\gamma(\alpha)})$ and $A \notin R_{\leq\gamma(\alpha)}$. It follows by Definition 4.3 that $R_{\leq\gamma(\alpha)} \cup F \cup \{A\}$ is inconsistent, so is $R'_{\leq\alpha} \cup F \cup \{A\}$. Thus R'_α is maximal.

"$\Leftarrow$": For every $\Gamma \in K \Downarrow F$, we prove that $E = Cn(\Gamma \cup F)$ is an extension of Δ. Let $R_\alpha = \Gamma \cap Cn(D_{\leq\alpha})$. To show that for all $\alpha < \eta$, R_α is a maximal subset of $Cn(D_{\leq\alpha})$ such that $R_{\leq\alpha} \cup F$ is consistent, assume that $A \in Cn(D_{\leq\alpha})$ and $A \notin \bar{R}_\alpha$. Let $K_{\leq\beta_\alpha} = Cn(D_{\leq\alpha})$ where β_α is the smallest ordinal β such that $K_{\leq\beta} = Cn(D_{\leq\alpha})$. Then we have that $A \in K_{\leq\beta\alpha}$ and $R_\alpha = \Gamma \cap Cn(D_{\leq\alpha}) = \Gamma \cap K_{\leq\beta_\alpha} = \Gamma_{\leq\beta_\alpha}$. So $A \notin R_\alpha$ implies that $A \notin \bar{\Gamma}_{\leq\beta_\alpha}$. It follows by $\Gamma \in \bar{K} \Downarrow F$ that $\Gamma_{\leq\beta_\alpha} \cup F \cup \{A\}$ is inconsistent, so is $R_\alpha \cup F \cup \{A\}$, as desired. □

The following corollary shows that the two formalizations above are identical in a very special case.

Corollary 4.1. *Let $\Delta = (D, F)$ be a WOP DT with a WOP $\Sigma = (D, \mathcal{P}, <)$. If $D \equiv Cn(D)$ and Σ is a POP, then*

E is a syntax-dependent extension of Δ iff E is an extension of Δ.

Proof. It follows directly from $\Sigma = \Sigma^K$ and the proof of Theorem 4.1. □

4.3 Default Reasoning with Constraints

Poole[4] added a set of constraints to his framework in order to block unwanted applications of defaults. Nebel[9] extended Poole's default theories with constraints into ranked default theories with constraints and discussed relationships between them and prioritized removal operations. In this subsection we will add constraints to WOP DT and give an direct relationship between the WOP DT with constraints and the belief contraction.

Definition 4.4. *A WOP DT with constraints is a triple $\Delta = (D, F, C)$, where D, F, and C are sets of sentences and C is interpreted as constraints.*

The definition of extension is revised as follows:

Let $\Sigma = (D, \mathcal{P}, <)$ be a WOP of D and η be the order-type of $\mathcal{P}$.

An extension of Δ is a set of sentences, $E = Cn((\cup_{\alpha<\eta} R_\alpha) \cup F)$, such that for all $\alpha < \eta$, $R_\alpha \subseteq Cn(D_{\leq\alpha})$ and R_α is a maximal subset of $Cn(D_{\leq\alpha})$ such that $R_{\leq\alpha} \cup F$ is consistent.

The notion of strong provability can be defined as usual.

Theorem 4.2. *Let $\Delta = (D, F, C)$ be a WOP DT with constraints and $\Sigma^D = (D, \mathcal{P}^D, <^D)$ be a WOP of D. Let $K = Cn(D)$ and $\Sigma^K = (K, \mathcal{P}^K, <^K)$ be the POP induced by Σ^D. Then for all sentences A in L,*

$$\Delta \mid\!\sim A \text{ iff } A \in K \ominus (F \cup C) + F$$

Proof. Similar to the proof of Theorem 4.1. □

As a special case of the theorem above, we have:

Corollary 4.2. *Let $\Delta = (D, \{\}, C)$ be a WOP DT with constraints C and $\Sigma^D = (D, \mathcal{P}^D, <^D)$ be a WOP of D. Let $K = Cn(D)$ and $\Sigma^K = (K, \mathcal{P}^K, <^K)$ be the POP induced by Σ^D. Then for all sentences A in L,*

$$\Delta \mid\!\sim A \text{ iff } A \in K \ominus C$$

This result gives an answer to the question of whether we may usefully introduce into the nonmonotonic contexts some sort of counterpart to contraction, raised by Makinson and

Gärdenfors[6], whih is still considered to be an interesting open question in [9][3]. The above corollary tells us that facts-free default theory with constraints is strictly correspondent to belief contraction in the generalized framework.

5 Related Work and Conclusion

As mentioned above, there is much work in the literature[3,5,6,12–15] concerning the relationship between nonmonotonic reasoning and belief revision. In [6], Makinson and Gärdenfors investigated the relation between singleton belief revision and nonmonotonic logic. They introduced a translation from the membership of belief revision, $y \in K * x$, into the inference relation of nonmonotonic reasoning $x \mid\!\sim y$. The correspondence is only in generalities because no specific nonmonotonic reasoning system was given which follows strictly this correspondnece. Their work, however, is pioneering. In [5] Brewka gave an equivalent relationship between his preferred sub-theories and his belief revision operation. But his belief revision operator is not the one of AGM's. Boutilier[12] presented a unified framework for default reasoning and belief revision. Being different from ours, his method is based on modal framework of default logic and belief revision.

In [16], Zhang *et al.* showed some features of the syntax-independent default reasoning. It was proved that the strong provability $\mid\!\sim$ is a finite supracompact rational nonmonotonic inference relation, that is to say, $\mid\!\sim$ satisfies the following rules:

(RN1) If $\Gamma \vdash A$, then $\Gamma \mid\!\sim A$ (Supraclassicality).

(RN2) If $\Gamma \mid\!\sim \bot$, then $\Gamma \vdash \bot$ (Consistency Preservation).

(RN3) If $\Gamma \mid\!\sim \Delta \vdash A$, then $\Gamma \mid\!\sim A$ (Closure or Weak Transitivity).

(RN4) If $\Gamma \cup \Delta \mid\!\sim A$ and $\Delta \neq \phi$, then there are $A_1, \ldots, A_n \in \Delta$ such that $\Gamma \mid\!\sim (A_1 \wedge \cdots \wedge A_n) \rightarrow A$ (Infinite Conditionalization).

(RN5) If $\Gamma \not\mid\!\sim \neg(A_1 \wedge \cdots \wedge A_n)$ for all $A_1, \ldots, A_n \in \Delta$, then $\Gamma \mid\!\sim A$ implies $\Gamma \cup \Delta \mid\!\sim A$ (Infinite Rational Monotonicity).

(RN6) $\Gamma \mid\!\sim A$ iff there exists a finite subset Γ_0 of Γ such that $\Gamma_0 \cup \Gamma' \mid\!\sim A$ for every finite subset Γ' of $Cn(\Gamma)$ (Finite Supracompactness).

In this paper we extended Nebel's ranked default theory in two ways: syntax-dependent approach and syntax-independent approach. We showed that the syntax-independent default reasoning is strictly correspondent to the general belief revision operation. This result was further extended to the case of default theories with constraints. We showed that the facts-free default theory with constraints is a sort of counterpart of contraction in the nonmonotonic reasoning contexts.

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