Capacity Allocation with Competitive Retailers

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ABSTRACT

This paper addresses a problem in supply chain management that how scarce resources can be efficiently allocated among competing interests. We present a formal model of allocation mechanisms for such settings that a supplier with limited production capacity allocates its products to a set of competitive retailers. In contrary to the existing allocation mechanisms in which retailers are local monopolists, the new model exhibits much more complicated market behaviors. We show that the widely-used proportional allocation mechanism is no longer necessarily Pareto optimal, even if all retailers are in a symmetric situation. A necessary and sufficient condition for the proportional allocation to be Pareto optimal is given. We propose a truth-inducing allocation mechanism based on our capacity allocation model, which is more intuitive and applicable than the existing truth-inducing mechanisms.

Categories and Subject Descriptors

K.4 [**Computers and Society**]: Electronic Commerce; I.2.1 [**Artificial Intelligence**]: Applications and Expert Systems—*Games*

General Terms

Design, Economics, Management

Keywords

Allocation mechanism design, game theory, oligopoly

1. INTRODUCTION

Mechanism design has been an active research area in economics, management science and operations research for over twenty years, and recently becomes an emerging research topic in computer science and e-business[7, 9, 13, 16]. Such a research has resulted many applications in many areas such as resource allocation, supply chain management, and e-marketplace development[2, 3, 9, 14].

In many industries a market supplier often confronts the situation that its total demand from retailers she supplies exceeds its production capacity due to uncertainty of market demands, costly capacity construction and time consuming capacity expansion. In such a case, retailers tend to order more than they need in order to receive more favorable allocation. This makes the market to be unstable and malfunctioning. Such a problem is known as the capacity allocation problem[5]. One possible solution to the problem is to design an appropriate allocation mechanism to reduce the competition among retailers. A set of such a mechanism have been devised and some of them have been generally used in the industry, such as proportional allocation, linear allocation, channel member selection and auction [5, 12, 8]. In particular, allocation mechanisms have been employed in industries such as semiconductor [8, 15], automobile [4] and telecommunication [11]. However, the efficiency of these allocation mechanisms heavily relies on market formations.

In [5] Cachon and Lariviere investigate the properties of capacity allocation mechanisms for the markets where a single supplier supplies multiple numbers of retailers who enjoy local monopoly (exclusive distribution). It is shown that the proportional allocation mechanism, one of the commonly used mechanisms in industry, maximizes the sum of retailer profits assuming all truthfully submit their optimal orders. However, the result is built upon the assumption that there is no competition in the downstream market (retail market). Obviously, in the more general and complex situations when the retailers have direct competition in retail market, the market behaviors and the properties of allocation mechanisms would be significantly different. In this paper we will relax Cachon and Lariviere's assumption and investigate the properties of capacity allocation mechanisms in the situations when competition exists in retail market. We assume that all retailers compete with each other for customer orders in an oligopolistic market. As we will see, the allocation mechanisms exploited in the upstream market (wholesale market) are highly related to the mechanisms used in the downstream market. For instance, the typical capacity allocation mechanism, *proportional allocation*, is no longer Pareto optimal whilst it is so in Cachon and Lariviere's model. The other mechanism properties, such as *individual rationality* and *incentive compatibility*, are more closed to the real market situations.

This paper is organized as follows. In Section 2, we describe a structure and rules of our model, and criteria of allocation mechanism design. In Section 3, we model the downstream market as a standard Cournot game. In Section 4, we consider properties of proportional allocation according to supply chain members' perspectives. In Section 5, we propose a new truth-inducing allocation mechanism. Finally, in Section 6, we conclude the paper and discuss the related work.

2. CAPACITY ALLOCATION MECHANISMS

Capacity allocation problem can be abstracted as a three-tier supply chain: *a single supplier*, *a number of retailers*, and *a range of end-users*. It is assumed that all products of the supplier are sold through retailers to the end-users. Figure 1 depicts the organization of the supply chain.

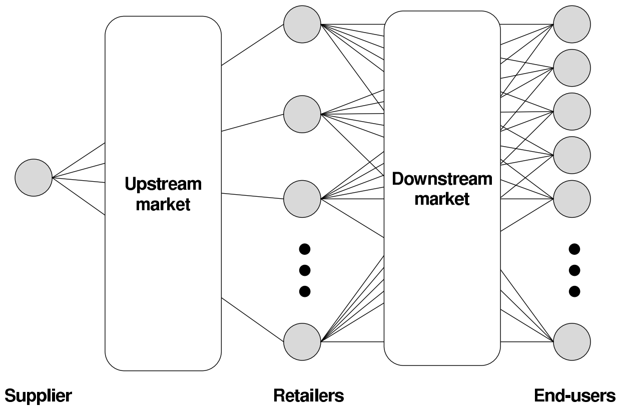

Figure 1: Supply Chain Model

If we view the whole supply chain as a single market, the supplier will be a monopolist who dominates the market. Given the total market demand, it is easy for the supplier to determine the optimal market supply, known as *the monopolist's quantity*, by applying the standard analysis of market demand and supply (see [16]p.385). Once the monopolist's quantity is settled, the supply of the products under consideration will become a bounded resource to the retailers in the *upstream market* shown in the above figure. On the other hand, in the case when the production capacity of the supplier is no more than the monopolist's quantity, the total market supply will be the minimum of the monopolist's quantity and its production capacity. Therefore, the sales of the products in the upstream market are actually a process of capacity allocation. The main problem of supply chain management is then to find an appropriate market mechanism under which the products produced by the supplier can be efficiently allocated to all the retailers. Note that we cannot simply apply the price leverage in the upstream market, because the profit of the supplier is co-effected by both the upstream market and the downstream market. In other words, the efficiency of capacity allocation mechanism applied in the upstream market depends on the market mechanism used in the downstream market. In [6], Cachon and Lariviere analyze a few capacity allocation mechanisms based on the assumption that each retailer enjoys its local monopoly. In other words, there is no competition between retailers. This is applicable to a few industries, such as car services, fast-food restaurant chains and other monopolies of sales. However, for most industries, severe competition between retailers exists. Such industries include ISP services, mobile communication, online sales, and so on. In this paper, we analyze the capacity allocation problem in the situation where retailers compete with each other in the downstream market. We examine how allocation mechanisms chosen by the supplier influence retailers' order quantity. In contrast to Cachon and Lariviere's approach, we assume that the downstream market is characterized by Cournot oligopolistic model, so only quantity competition is concerned (see [16]p.240).

2.1 Capacity allocation game

We consider a single-period game in which a single supplier sells a single type of product to $n \geq 2$ retailers. All the retailers compete with each other in an oligopolistic market to resale products to end-users. During the game, the following sequence of events occurs. First, the supplier decides its production capacity K of the product whose construction cost is c per unit. We assume that the capacity is not changeable during the game. Second, the supplier chooses allocation mechanism g(see the following definition) and notifies the mechanism to retailers. Third, all retailers learn the end-user market demand θ. Based on both the market demand and the notified allocation mechanism, retailer i determines its order quantity m_i in order to earn its profit π_i. All retailers simultaneously submit purchase orders to the supplier. Fourth, the supplier produces quantity Q, all of which are supplied to the retailers. Supplying to each retailer at a fixed wholesale price w is according to allocation mechanism g and retailers orders $m = (m_1, \cdots, m_n)$. Finally, retailers sell allocated products in the same downstream market whose demand function is F. The market price is determined by the inverse demand function P. We assume that a disposal cost of the product is expensive. Hence, retailers do not have a policy to sustain market price by not selling all inventories. We refer to this type the game as a *capacity allocation game*. It is easy to see that our definition of capacity allocation game only differs from the one given in [5] is a structure of the downstream market. While Cachon and Lariviere assume all retailers enjoy local monopolies without direct competition, we assume that all retailers are in a single oligopolistic market with direct competition.

Let $\mathcal{A} = \{a \in \Re^n : a \geq 0 \;\;\&\;\; \sum_{i=1}^{n} a_i \leq K\}^1$. We call each $a \in \mathcal{A}$ as a *feasible allocation*.

Definition 1. An *allocation mechanism* is a function $g : \Re^n \to \mathcal{A}$ which assigns a feasible allocation to each vector of orders such that for any retailers' order vector m, $g_i(m) \leq m_i$ for each $i = 1, \cdots, n$.

2.2 Criteria of allocation mechanism design

Main concerns for mechanism design are stability and efficiency. Three typical criteria are generally applied to mechanism design: *Incentive Compatibility*, *Individual Rationality* and *optimality* [16]. The first criterion restricts our attention to the mechanisms in which truth telling is an optimal strategy for each agent. In the setting of capacity allocation mechanisms, it can be described as follows:

[1] A vector $a \geq 0$ means for any component a_i of the vector, $a_i \geq 0$.

Definition 2. An allocation mechanism g is called to be *incentive compatible* (IC) or *truth-inducing* if all retailers placing orders truthfully at their optimal sales quantities is a Nash equilibrium of g.

Note that the retailers' profits rely on the model of the downstream market. We describe this point in the next section.

The principle of individual rationality in the setting of capacity allocation can be described as follows:

Definition 3. An allocation mechanism g is called to be *individually responsive* (IR) if for any i,

$$m_i' > m_i \text{ implies } g_i(m_i', m_{-i}) > g_i(m_i, m_{-i}) \quad \text{unless } g_i(m) = K. \tag{1}$$

An allocation mechanism g is called to be *weakly individually responsive* (WIR) if for any i,

$$m_i' > m_i \text{ implies } g_i(m_i', m_{-i}) \geq g_i(m_i, m_{-i}), \tag{2}$$

where m is a vector of retailers' orders, m_i is the i's component of m, m_{-i} is a vector of the other retailers' orders, and m_i' is a variation of m_i.

In other words, under an IR mechanism, a retailer receives more allocation, if it places more order. If all retailers are local monopolists, their profits will be linearly increasing with the increase of sales quantity. Therefore, whenever retailers expect that the supplier does not have enough capacity, they place more orders than optimal order quantity to pull out the optimal allocation from the supplier. This means that the two criteria described above are essentially incompatible under the assumption that the downstream markets are locally monopolistic. This depravation has been partially verified by Cachon and Lariviere in [5].

A retailers' order vector m is called to be a dominant equilibrium if for each retailer i, m_i maximizes its profit, regardless of orders of the other retailers. Cachon and Lariviere prove that the truth-telling is not dominant strategy equilibrium under IR mechanism.

Theorem 1. [5] Under the assumption that the downstream markets are locally monopolistic, all retailers truthful orders at their optimal sales quantities is not a dominant equilibrium under any individually responsive allocation mechanisms.

Note that if there is a direct competition in the downstream market, the above claim will not be true. We describe this topic in Section 5.

Before we complete the section, let us consider two examples of allocation mechanisms.

An allocation mechanism g is *proportional allocation* if

$$g_i(m) = min\left\{m_i, \frac{m_i}{\sum_{j=1}^{N} m_j} K\right\} . \tag{3}$$

In other words, whenever capacity binds, allocated quantity to each retailer is the same fraction of its order under the proportional allocation. Obviously the proportional allocation is an IR allocation.

An allocation mechanism g is *uniform allocation* if

$$g_i(m, \hat{n}) = \begin{cases} \frac{1}{\hat{n}}(K - \sum\limits_{j=\hat{n}+1}^{n} m_j), & i \leq \hat{n}, \\ m_i, & i > \hat{n}, \end{cases} \tag{4}$$

where the retailers' orders are sorted in decreasing order and $\hat{n}$ is the largest integer less than or equal to n such that $g_n(m,^n) \leq m_n$. Since uniform allocation always favors small retailers, it is proved that the uniform allocation is a truth-inducing mechanisms[17].

Another criterion we consider for allocation mechanism design is whether a mechanism can maximize the expected revenue - the sum of profits of supply chain members - among all mechanisms that are incentive compatible and/or individual responsive.

Definition 4. An allocation mechanism g is *Pareto allocation mechanism* if it maximizes the sum of retailers' profits assuming all retailers truthfully submit their optimal orders.

In [5], Cachon and Lariviere show that in the case when all retailers in the downstream market of an allocation game enjoy local monopolies, the proportional allocation is actually a Pareto allocation mechanism. However, this is no longer true in our model.

3. DOWNSTREAM MARKET MODEL

As indicated in Subsection 2.1, the downstream market of the capacity allocation game we consider is a typical oligopoly, so it can be characterized by the classical Cournot model[16, 18]. In this section, we present the model of the downstream market and investigate its interrelationship to the upstream market in the next section.

Assume that F is the end-users' aggregated demand function which is strictly decreasing with the price of the product. The price function $P(\bullet)$ can then be described as the inverse function of F,

$$P(Q) = F^{-1}(Q),$$

where Q is total supply to the downstream market. For each retailer i, if C_i is its cost function of sales, its profit can then be calculate by the following formula,

$$\pi_i(q_i) = P(Q)q_i - C_i(q_i) \ , \tag{5}$$

where q is a retailers' sales quantity vector. At the *Cournot-Nash equilibrium* equilibrium, $q^* = (q_1^*, q_2^*, \ldots, q_n^*)$, each retailer's profit is maximized, i.e.,

$$\forall q_i \in \Re^+ (\pi_i(q^*) \geq \pi_i(q_i, q_{-i}^*)) \quad (i = 1, \cdots, n) \tag{6}$$

We assume that the profit function $\pi_i(q)$ is concave, and the function $P(\bullet)$ and $C_i(\bullet)$ are differentiable. Then, Equation (6) implies the following first-order conditions:

$$P'(Q^*)q_i^* = C_i'(q_i^*) - P(Q^*) \qquad (i = 1, \cdots, n), \tag{7}$$

where $f'(\bullet)$ stands for the derivative of the function f.

Suppose that there is no fixed cost for each retailer and the price function $P(\bullet)$ is linear. Then, the cost function and the price function can be simplified as follows:

$$Ci(q_i) = wq_i, \qquad w > 0,$$
$$P(Q) = \max\{\alpha - \beta Q, 0\},$$

where $\alpha > w \geq 0$ and $\beta > 0$ are constants, and w is a purchase price per unit which is the supplier's wholesale price. In such a case, the condition (7) induces

$$\alpha - \beta(q_1^* + q_2^* + \ldots + q_n^*) - w = \beta q_i^*.$$

Its solution, Cournot-Nash equilibrium is

$$q_i^* = \frac{\alpha - w}{(n+1)\beta} \quad (i = 1, 2, \ldots, n). \tag{8}$$

4. PARETO OPTIMALITY

Based on the above modeling, from this section, we investigate the properties of our allocation game. In this section, we explore how the market mechanism of the downstream market influences all of supply chain members' profits - Pareto optimality of allocation mechanisms.

4.1 Supplier's Profit Maximization

In [5], Cachon and Lariviere show that in the case when all retailers in the downstream market of an allocation game enjoy local monopolies and submit truthful orders, the proportional allocation maximizes both the supplier's profit and the sum of retailers' profits. In this sense, the proportional allocation can be considered the best capacity allocation mechanism in the monopolistic downstream market allocation game. However, this is no longer true in the oligopolistic downstream market allocation game. To show this difference, we first prove that the proportional allocation mechanism does maximize the supplier's profit, if all retailers order truthfully their optimal sales quantity.

According to Section 3, the equilibrium sales quantity of each retailer is $\frac{\alpha-w}{(n+1)\beta}$. Therefore, if each retailer orders truthfully its optimal sales quantity, the retailers' order vector m will be

$$m_i^* = q_i^* = \frac{\alpha - w}{(n+1)\beta} \quad (i = 1, 2, \ldots, n).$$

Lemma 1. Proportional allocation with the oligopolistic downstream market maximizes the supplier's profit π_s, if all retailers submit true orders m_i^* for all i.

PROOF. According to the definition of our capacity allocation game, the supplier's profit π_s can be calculated as follows:

$$\pi_s(Q) = wQ - cK \ ,$$

where Q is the supplier's total supply and $Q \leq K$.

In case the supplier's capacity binds, we have $K \leq \sum_{j=1}^{n} m_j^* = \frac{n(\alpha-w)}{(n+1)\beta}$. According to the definition of the proportional allocation, $g_i(m^*) = \frac{K}{n}$ $(i = 1, \cdots, n)$. Thus $Q = \sum_{j=1}^{n} g_j(m^*) = K$. This means the proportional allocation maximizes the supplier's profit. In case there is enough capacity, the sum of the proportional allocation is $\sum_{i=1}^{n} m_i^*$ and the feasible total allocation quantity is $\sum_{i=1}^{n} g_i(m^*) \leq \sum_{i=1}^{n} m_i^*$. Hence, in both case, the proportional allocation maximizes the supplier's profit. □

Lemma 1 shows that the proportional allocation is the best allocation mechanism from the supplier's perspective.

4.2 Retailers' Profit Maximization

We have seen that the proportional allocation mechanism is in favor of the supplier. We now analyze its properties from the retailers' point of view.

Theorem 2. If all retailers order truthfully Cournot-Nash equilibrium quantity $m_i^* = \frac{\alpha-w}{(n+1)\beta}$, the proportional allocation mechanism is Pareto optimal if and only if $K \leq \frac{\alpha-w}{2\beta}$.

PROOF. The optimization problem of total retailers' profits is formulated as follows:

$$\max_{a\in\mathcal{A}} \sum_{i=1}^{n} \pi_i(a) = \max_{a\in\mathcal{A}} \sum_{i=1}^{n} \left(\left(\alpha - \beta \sum_{l=1}^{n} a_l \right) a_i - w a_i \right)$$

s.t.

$$\sum_{i=1}^{n} a_i \leq K \tag{9}$$

$$a_i \leq m_i^* \qquad \text{for } i = 1, 2, \ldots, n \tag{10}$$

$$a_i \geq 0 \qquad \text{for } i = 1, 2, \ldots, n$$

$$\lambda_j \geq 0 \qquad \text{for } j = 1, 2, \ldots, N$$

where λ is Karush-Kuhn-Tucker (KKT) coefficents, j is a number of inequality conditions, and $N = n + 1$ from Equation (9) and (10). The optimal solution a^* of $\sum_{i=1}^{n} \pi_i(a)$ satisfies the KKT conditions (see [10]p.582) which are,

$$\left(\alpha - w - 2\beta \sum_{l=1}^{n} a_l^* \right) - (\lambda_1 + \lambda_{i+1}) \geq 0, \qquad \text{for } i = 1, \ldots, n,$$

$$a_i^* \left(\alpha - w - 2\beta \sum_{l=1}^{n} a_l^* - (\lambda_1 + \lambda_{i+1}) \right) = 0, \qquad \text{for } i = 1, \ldots, n, \tag{11}$$

$$\sum_{i=1}^{n} a_i^* - K \leq 0, \qquad \text{for } j = 1, \tag{12}$$

$$a_{j-1}^* - m_{j-1}^* \leq 0, \qquad \text{for } j = 2, \ldots, N, \tag{13}$$

$$\lambda_1 \left(\sum_{i=1}^{n} a_i^* - K \right) = 0, \qquad \text{for } j = 1, \tag{14}$$

$$\lambda_j \left(a_{j-1}^* - m_{j-1}^* \right) = 0, \qquad \text{for } j = 2, \ldots, N,$$

$$a_i^* \geq 0, \qquad \text{for } i = 1, \ldots, n, \tag{15}$$

$$\lambda_j \geq 0, \qquad \text{for } j = 1, \ldots, N. \tag{16}$$

In the case of $\lambda_1 = 0, \ldots, \lambda_{n+1} = 0$, we derive following conditions. According to Equation (11), we have,

$$\sum_{i=1}^{n} a_i^* = \frac{\alpha - w}{2\beta}.$$

By putting Equation (11) and (12), we yield,

$$K \geq \frac{\alpha - w}{2\beta}.$$

From Equation (13) and (15), we obtain,

$$0 \leq a_i^* \leq m_i^*, \quad \text{for } i = 1, \ldots, n.$$

Thus, if $K \geq \frac{\alpha-w}{2\beta}$ and $0 \leq a_i^* \leq m_i^*$ (for $i = 1, \ldots, n$), the total retailers' profit $\Pi_r(a)$ is maximised at a^*,

$$\Pi_r(a^*) = \sum_{i=1}^{n} \left(\left(\alpha - \beta \sum_{l=1}^{n} a_l \right) a_i - w a_i \right), = \frac{(\alpha - w)^2}{4\beta},$$

where a^* satisfies $\sum_{i=1}^{n} a_i^* = \frac{\alpha-w}{2\beta}$.

From Equation (3), the total retailers' profits under proportional allocation with enough production capacity is,

$$\begin{aligned}\Pi_r(g_e(m^*)) =& \sum_{i=1}^{n}\left(\left(\alpha-\beta\sum_{l=1}^{n} m_l^*\right)m_i^* - wm_i^*\right),\\ =& \frac{n}{(n+1)^2}\frac{(\alpha-w)^2}{\beta},\end{aligned}$$

where $g_e(m^*)$ is a vector of allocated quantity under proportional allocation with enough capacity as follows,

$$g_e(m^*) = \left(\frac{n(\alpha-w)}{(n+1)\beta},\dots,\frac{n(\alpha-w)}{(n+1)\beta}\right).$$

On the other hand, if the production capacity is scarce ($\frac{\alpha-w}{2\beta} < K < \frac{n}{n+1}\frac{(\alpha-w)}{\beta}$),

$$\begin{aligned}\Pi_r(g_s(m^*)) =& \sum_{i=1}^{n}\left(\left(\alpha-\beta\sum_{l=1}^{n}\frac{K}{n}\right)\frac{K}{n} - w\frac{K}{n}\right),\\ =&(\alpha-\beta K-w)K,\end{aligned}$$

where $g_s(m^*)$ is a vector of allocated quantity under proportional allocation with scarce capacity as follows,

$$\begin{aligned}g_s(m^*) =& \left(\frac{m_i^*}{\sum_{i=1}^{n} m_i^*}K,\dots,\frac{m_i^*}{\sum_{i=1}^{n} m_i^*}K\right),\\ =& \left(\frac{K}{n},\dots,\frac{K}{n}\right).\end{aligned}$$

$\Pi_r(g_s(m^*))$ is convex and it is maximized at $K = \frac{\alpha-w}{2\beta}$. Therefore, $\Pi_r(a^*) > \Pi_r(g_s(m^*))$. Hence, if $K > \frac{\alpha-w}{2\beta}$, $\Pi_r(a^*)$ is greater than the profit under proportional allocation,

$$\begin{aligned}\Pi_r(a^*) =& \frac{(\alpha-w)^2}{4\beta}\\ >& \begin{cases}\Pi_r(g_s(m^*)) = (\alpha-\beta K-w)K, \\ \qquad \frac{\alpha-w}{2\beta} < K < \frac{n}{n+1}\frac{(\alpha-w)}{\beta},\\ \Pi_r(g_e(m^*)) = \frac{n}{(n+1)^2}\frac{(\alpha-w)^2}{\beta},\\ \qquad \frac{n}{n+1}\frac{\alpha-w}{\beta} \le K.\end{cases}\end{aligned}$$

Thus, proportioanl allocation is not the total retailers' profit maximizing mechanism, if $K > \frac{\alpha-w}{2\beta}$.

In the case $\lambda_1 \neq 0, \lambda_2 = 0, \dots, \lambda_{n+1} = 0$, we derive following conditions,

$$\begin{aligned}&\alpha - w - 2\beta\sum_{l=1}^{n} a_l^* = \lambda_1 \ge 0,\\ &\frac{\alpha-w}{2\beta} \ge \sum_{l=1}^{n} a_l^* = \sum_{i=1}^{n} a_i^*,\\ &\frac{\alpha-w}{2\beta} \ge K.\end{aligned}$$

According to Equation (14), we obtain,

$$\sum_{i=1}^{n} a_i^* = K. \tag{17}$$

From Equation (11), we have,

$$a_i^*\left(\alpha - w - 2\beta\sum_{l=1}^{n} a_l^* - \lambda_i\right) = 0, \qquad \text{for } i = 1,\dots,n. \tag{18}$$

By putting Equation (17) and (18) together, we yield,

$$\alpha - w - 2\beta\sum_{i=1}^{n} a_i^* = \lambda_1. \tag{19}$$

From Equation (16) and (19), we have,

$$\alpha - w - 2\beta\sum_{l=1}^{n} a_l^* = \lambda_1 \ge 0. \tag{20}$$

By using Equation (20), we obtains,

$$\frac{\alpha-w}{2\beta} \ge \sum_{l=1}^{n} a_l^* = \sum_{i=1}^{n} a_i^*. \tag{21}$$

From Equation (17) and (21), we yield,

$$\frac{\alpha-w}{2\beta} \ge K.$$

Thus, if $K \le \frac{\alpha-w}{2\beta}$ and $0 \le a_i^* \quad (for\ i = 1,\dots,n)$, then $\sum_{i=1}^{n} a_i^* = K$ which is equivalent to allocated quantity under proportional allocation. Rest of the cases are redundant. □

Notice that the equilibrium order quantity for each retailer is $\frac{\alpha-w}{(n+1)\beta}$. So the total market demand for the downstream market is $\frac{n(\alpha-w)}{(n+1)\beta} \simeq \frac{\alpha-w}{\beta}$ when n (number of retailers) is reasonably big. Theorem 2 indicates that only if the supplier's production capacity is severely scarce (can only meets no more than half of the demand), the proportional allocation maximizes retailers' profits, otherwise retailers' profits are not maximized.

On the other hand, in case the capacity is greater than $\frac{\alpha-w}{2\beta}$, the total retailers' profits are maximized at the total allocation quantity $\frac{\alpha-w}{2\beta}$ which is exactly same quantity as the optimal sales quantity, when the supplier dominates the whole downstream market as a monopolist. Therefore, the optimal allocation quantity $\frac{\alpha-w}{2\beta}$ for the retailers has a same effect for retailers when all retailers cooperatively submit *the cartel quantity* whose sum is $\frac{\alpha-w}{2\beta}$.

In the real business, there is normally no allocation mechanism which satisfies all the supply chain members. Negotiations are required for balancing the benefit of market participants. We have seen that the proportional allocation mechanism is in favor of the market supplier. By Theorem 2, we can easily devise a variation of proportional allocation mechanism which maximizes the retailers' profits.

We call the mechanism as *cartel-based proportional allocation*,

$$g_i(m) = \begin{cases} min\left\{m_i, \frac{m_i}{\sum_{j=1}^{n} m_j}\frac{\alpha-w}{2\beta}\right\}, & K > \frac{\alpha-w}{2\beta},\\ min\left\{m_i, \frac{m_i}{\sum_{j=1}^{n} m_j}K\right\}, & K \le \frac{\alpha-w}{2\beta}.\end{cases} \tag{22}$$

It is easy to prove that the cartel-based proportional allocation is Pareto optimal. Notice that the allocation mechanism requires that the market information (α and β) of the oligopolistic downstream market is common knowledge.

While the cartel-based proportional allocation maximizes the total retailers' profits, it no longer maximizes the supplier's profit in the case that $\sum_{i=1}^{n} m_i > \frac{\alpha-w}{2\beta}$ and $K > \frac{\alpha-w}{2\beta}$. In such a case, whenever $\sum_{i=1}^{n} m_i \ge K$, the suppliers' profit

π_s is,

$$\begin{aligned}\pi_s &= w\sum_{i=1}^{n} g_i(m) - cK,\\ &= w\sum_{i=1}^{n}\left(\frac{m_i}{\sum_{j=1}^{n} m_j}\frac{\alpha-w}{2\beta}\right) - cK,\\ &= w\frac{\alpha-w}{2\beta} - cK,\\ &< wK - cK.\end{aligned}$$

When $\sum_{i=1}^{n} m_i < K$ but $\sum_{i=1}^{n} m_i > \frac{\alpha-w}{2\beta}$, the suppliers' profit π_s is,

$$\begin{aligned}\pi_s &= w\sum_{i=1}^{n} g_i(m) - cK,\\ &= w\sum_{i=1}^{n}\left(\frac{m_i}{\sum_{j=1}^{n} m_j}\frac{\alpha-w}{2\beta}\right) - cK,\\ &= w\frac{\alpha-w}{2\beta} - cK,\\ &< w\sum_{i=1}^{n} m_i - cK.\end{aligned}$$

Therefore, the outcome of the negotiation between the supplier and the retailers can range from the original proportional allocation to the cartel-based proportional allocation.

5. TRUTH-INDUCING MECHANISMS

In the previous section, we focus on the profit optimality of allocation mechanisms. Now we switch to the other two criteria of allocation mechanism design: *incentive compatibility* and individual responsiveness. As we have mentioned in Section 2.2, IC and IR are the properties we cannot have both, especially when the downstream market is monopolistic. In this section, we show that we could have both IC and the weak version of IR with the oligopolistic downstream market.

Several IC or truth-inducing mechanisms, such as uniform allocation, lexicographic allocation, and relaxed linear allocation are proposed in the literature [6, 5, 17]. However, such allocation mechanisms are not commonly used in industry, due to their non-intuitiveness. In this section, we propose a much more intuitive truth-inducing mechanism.

Definition 5. Given a vector $\gamma \in \Re^+$ such that $\sum_{i=1}^{n} \gamma_i \leq K$, a capacity allocation mechanism g is called as *capped allocation* if

$$g_i(m) = min\{m_i, \gamma_i\}, \quad i = 1, \cdots, n.$$

If $\gamma_1 = \cdots = \gamma_n$, we call such an allocation mechanism as *uniformly capped allocation*.

It is easy to see that any capped allocation is weakly individually responsive. The following theorem shows that any uniformly capped allocation is also truth-inducing.

Theorem 3. Any uniformly capped allocation mechanism with the oligopolistic downstream market is truth-inducing.

PROOF. Since the equilibrium sales quantity for each retailer i is $q_i^* = \frac{\alpha-w}{(n+1)\beta}$, its truthful order is $m_i^* = \frac{\alpha-w}{(n+1)\beta}$. Then it is sufficient to show that for any m_i,

$$\pi_i(g_i(m_i^*, m_{-i}^*)) \geq \pi_i(g_i(m_i, m_{-i}^*)),$$

where $m_i^* = \frac{\alpha-w}{(n+1)\beta}$. From Equation (5), we have,

$$\begin{aligned}&\pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right)\\ &= \left(\alpha - \beta\sum_{j=1}^{n} g_j\left(m_i, m_{-i}^*\right)\right) g_i\left(m_i, m_{-i}^*\right) - w g_i\left(m_i, m_{-i}^*\right),\\ &= \left(\alpha - \beta\left(\sum_{j\neq i} g_j(m_i, m_{-i}^*) + g_i(m_i, m_{-i}^*)\right) - w\right) g_i\left(m_i, m_{-i}^*\right),\\ &= \left(\alpha - w - \beta\sum_{j\neq i} g_j(m_i, m_{-i}^*)\right) g_i(m_i, m_{-i}^*) - \beta g_i^2(m_i, m_{-i}^*).\end{aligned}$$

To calculate the formula, we consider the following two cases:

Case 1: $\gamma \geq \frac{\alpha-w}{(n+1)\beta}$,

$$\begin{aligned}&\pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right)\\ &= \left(\alpha - w - \beta\frac{(n-1)(\alpha-w)}{(n+1)\beta}\right) g_i\left(m_i, m_{-i}^*\right)\\ &\quad - \beta g_i^2\left(m_i, m_{-i}^*\right),\\ &= (\alpha - w)\frac{2}{n+1} g_i\left(m_i, m_{-i}^*\right) - \beta g_i^2\left(m_i, m_{-i}^*\right).\end{aligned}$$

If $m_i \leq \gamma$, then $\pi_i\left(g_i\left(m_i^*, m_{-i}^*\right)\right) \geq \pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right)$. If $m_i > \gamma$, then $\pi_i\left(g_i\left(m_i^*, m_{-i}^*\right)\right) = (\alpha - w)\frac{2}{n+1}\gamma - \beta\gamma^2 < \pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right)$.

Case: 2 $\gamma < \frac{\alpha-w}{(n+1)\beta}$,

$$\begin{aligned}\pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right) &= (\alpha - w - (n-1)\beta\gamma)\, g_i\left(m_i, m_{-i}^*\right)\\ &\quad - \beta g_i^2\left(m_i, m_{-i}^*\right),\end{aligned}$$

$$\pi_i\left(g_i\left(m_i^*, m_{-i}^*\right)\right) = (\alpha - w - (n-1)\beta\gamma)\,\gamma - \beta\gamma^2.$$

If $m_i \geq \gamma$, then $\pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right) = \pi_i\left(g_i\left(m_i^*, m_{-i}^*\right)\right)$. If $m_i < \gamma$, then $\pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right) = (\alpha - w - (n-1)\beta\gamma)\, m_i - \beta m_i^2$. Hence,

$$\begin{aligned}&\pi_i\left(g_i\left(m_i^*, m_{-i}^*\right)\right) - \pi_i\left(g_i\left(m_i, m_{-i}^*\right)\right) =\\ &\quad (\alpha - w - (n-1)\beta\gamma)(\gamma - m_i) - \beta(\gamma^2 - m_i^2)\\ &= (\alpha - w - (n-1)\beta\gamma - \beta(\gamma + m_i))(\gamma - m_i)\\ &= (\alpha - w - n\beta\gamma - \beta m_i)(\gamma - m_i),\\ &> (\alpha - w - n\beta\gamma - \beta\gamma)(\gamma - m_i),\\ &= (\alpha - w - (n+1)\beta\gamma)(\gamma - m_i),\\ &> 0.\end{aligned}$$

□

Under uniformly capped allocation mechanism, instead of receiving inflated orders from the retailers, the supplier induces truthful orders from the retailers which is extremely important for the supplier to determine its production capacity and its selling price from a long-term perspective. In other words, by choosing truth-inducing mechanisms, the supplier gets truthful demand information of the downstream market which leads the supplier's secure decision-makings on capacity planning and sales planning. However, it is easy to see that uniformly capped allocation is not necessarily efficient in the sense that uniformly capped allocation does not guarantee to allocate all available capacity to retailers. It is a trade-off for the supplier to choose either efficient and profitable mechanisms such as proportional allocation or its variations, or informative truth-telling mechanisms such as uniformly capped allocation.

6. CONCLUSION AND RELATED WORKS

We have presented a formal model of capacity allocation mechanisms for the supply chains in which the downstream market is oligopolistic. Due to the competitive nature of oligopoly, the market behaviors of capacity allocation mechanisms become much more complicated than the model with the monopolistic downstream market. It is well known in a situation when all retailers are in symmetric monopolistic markets, proportional allocation mechanism maximizes both the supplier's profit and the retailers' profits. However we have shown that the proportional allocation mechanism is no longer necessarily Pareto optimal, even if all retailers face a symmetric situation in the oligopolistic downstream market. A necessary and sufficient condition for the proportional allocation to be Pareto optimal has been given. We have also proposed a truth-inducing allocation mechanism based on our capacity allocation model, which is more intuitive and applicable than the existing truth-inducing mechanisms, such as uniform allocation, relaxed linear allocation and lexicographic allocation.

This paper is closely related to capacity allocation game in a model with one supplier and n independent retailers enjoying local monopolies [5], a model with one supplier and two retailers [6], and an internal organization model with one factory and n product managers[8]. As far as we are aware, the present paper is the first work which deals with the capacity allocation problem with the oligopolistic downstream market. We have shown that the market behaviors of our allocation game are significantly different from the ones of the allocation game with the monopolistic downstream market.

One of direct applications of our model is that we can use the approaches developed in the paper to analyze the Trading Agent Competition Game of Supply Chain Management (TAC SCM)[1]. TAC SCM specifies a virtual three-tier supply chain of personal computer manufacturing and marketing with two marketplaces: *component market* and *end-product market*. The end-product market is a typical oligopoly and the component market is operated under a fixed capacity allocation mechanisms[19]. Therefore, the TAC SCM falls exactly in the domains of our allocation game. In last few years, TAC SCM experiences a number of difficulties in the supplier model, such as "day 0 procurement", "lottery effect of component supply" and "coverage of component market". The methodologies developed in this paper shed light on a throughout solution to these problems and the design of other supply chain management systems. We leave this research as our future work.

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