Logical Properties of Belief-Revision-Based Bargaining Solution*

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Abstract. This paper explores logical properties of belief-revision-based bargaining solution. We first present a syntax-independent construction of bargaining solution based on prioritized belief revision. With the construction, the computation of bargaining solution can be converted to the calculation of maximal consistent hierarchy of prioritized belief sets. We prove that the syntax-independent solution of bargaining satisfies a set of desired logical properties for agreement function and negotiation function. Finally we show that the computational complexity of belief-revision-based bargaining can be reduced to $\Delta_2^P[\mathcal{O}(\log n)]$.

Keywords: belief revision, bargaining theory, automated negotiation.

1 Introduction

Much recent research has shown that belief revision is a successful tool in modeling logical reasoning in bargaining and negotiation[2,4,5,14,15,16]. These studies have established a qualitative solution to bargaining problem, which differentiates them from the traditional game-theoretic solution[6,11]. In [16], Zhang *et al.* proposed an axiomatic system to specify the logical reasoning behind negotiation processes by introducing a set of AGM-like postulates[1]. In [4,5], Meyer *et al.* further explored the logical properties of these postulates and their relationships. In [16], Zhang and Zhang proposed a computational model of negotiation based on Nebel's syntax-dependent belief base revision operation[8] and discussed its game-theoretic properties and computational properties. It was shown that the computational complexity of belief-revision-based negotiation can be Π_2^P-hard. However, it is left unknown that how this computational model related to the axiomatic approach to negotiation. In this paper, we shall establish a relationship between these two approaches and reassess the computational complexity of belief-revision-based negotiation. In order to make two different modeling approaches comparable, we shall redefine the bargaining solution given in [16] based

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on the assumption that bargaining inputs are logically closed[1]. We then shown that the computation of agreements can be reduced to the construction of maximal consistent hierarchies of negotiation items. Based on this result, we show that the new definition of bargaining solution satisfies most of desired logical properties of negotiation. Finally we present a completeness result on computational complexity of belief-revision-based bargaining solution, which shows that the decision problem of bargaining solution is $\Delta_2^P[\mathcal{O}(\log n)]$-complete. This result significantly improves the result presented in [16].

Similar to the work in [16], we will restrict us to the bargaining situations within which only two agents are involved. We assume that each agent has a set of negotiation items, referred to as *demand set*, which is describable in a finite propositional language $\mathcal{L}$. The language is that of classical propositional logic with an associated consequence operation Cn in the sense that $Cn(X) = \{\varphi : X \vdash \varphi\}$, where X is a set of sentences. A set K of sentences is *logically closed* or called a *belief set* when $K = Cn(K)$. If X, Y are two sets of sentences, $X + Y$ denotes $Cn(X \cup Y)$.

Suppose that X_1 and X_2 are the demand sets of two agents. To simplify exploration, we will use X_{-i} to represent the other set among X_1 and X_2 if X_i is one of them.

2 Prioritized Belief Revision

Suppose that K is a belief set and $\preceq$ a pre-order[2]. We define recursively a hierarchy, $\{K^k\}_{k=1}^{+\infty}$, of K with respect to the ordering $\preceq$ as follows:

1. $K^1 = \{\varphi \in K : \neg\exists\psi \in K(\varphi \prec \psi)\}$; $T^1 = K \backslash K^1$.
2. $K^{k+1} = \{\varphi \in T^k : \neg\exists\psi \in T^k(\varphi \prec \psi)\}$; $T^{k+1} = T^k \backslash K^{k+1}$.

where $\varphi \prec \psi$ denotes $\varphi \preceq \psi$ and $\psi \not\preceq \varphi$. The intuition behind the construction is that each time collects all maximal elements and remove them from the current set.

We will write $K^{\leq l}$ to denote $\bigcup_{k=1}^{l} K^k$. The following lemma shows that the hierarchy can only be finite if $\preceq$ satisfies the following *logical constraint*:

(LC) If $\varphi_1, \cdots, \varphi_n \vdash \psi$, $\min\{\varphi_1, \cdots, \varphi_n\} \preceq \psi$.

It is easy to see that such an order can induce an AGM epistemic entrenchment and vice versa[3]. Therefore such an ordering will be referred to as a *epistemic entrenchment(EE) ordering*. The following lemma is easy to verify and will be used intensively throughout the paper.

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[1] This assumption is essential because Zhang and Zhang's construction of bargaining solution is syntax-dependent(logically equivalent inputs could result different outcomes). Without the assumption, even the most fundamental postulates, such as *Extensionality*, cannot be satisfied.

[2] A pre-order over a set we mean in this paper is a complete ordering over the set which is transitive and reflexive.

Lemma 1. *Let K be a belief set and $\preceq$ a pre-order over K which satisfies (LC), then*

1. *for any l, $K^{\leq l}$ is a belief set.*
2. *There exists a number N such that $K = \bigcup_{k=1}^{N} K^k$.*

Let O be any set of sentences in $\mathcal{L}$, we define the *degree of coverage* of O over K, denoted by $\rho_K(O)$, to be the greatest number l that satisfies $K^{\leq l} \subseteq O$.

It is well-known that an AGM belief revision operator can be uniquely determined by an epistemic entrenchment ordering. Similar to [16], we will define a belief revision function based on the idea of maximizing retainment of most entrenched beliefs.

By following the convention introduced by Nebel[8], for any belief set K, a set of sentences F, and an EE ordering $\preceq$ over K, we define $K \Downarrow F$ as follows: for any $H \in K \Downarrow F$,

1. $H \subseteq K$;
2. for all $k = 1, 2, \cdots$, $H \cap K^k$ is set-inclusion maximal among the subsets of K^k such that $\bigcup_{j=1}^{k} (H \cap K^j) \cup F$ is consistent.

We call $\otimes$ a *prioritized revision function* over $(K, \preceq)$ if it is defined as follows:

$$K \otimes F \stackrel{def}{=} \bigcap_{H \in K \Downarrow F} Cn(H) + F.$$

Lemma 2. [Nebel 1992] $\otimes$ *satisfies all AGM postulates.*

3 Belief-Revision-Based Bargaining Solution

Now we redefine the bargaining solution given in [16]. Different from their work, we will define a bargaining game as a pair of prioritized belief sets rather than a pair of prioritized belief bases. We will see that this change results a significant differences in logical properties.

Definition 1. *A bargaining game is a pair of prioritized belief sets* $((K_1, \preceq_1), (k_2, \preceq_2))$*, where K_i is a belief set in $\mathcal{L}$ and $\preceq_i$ $(i = 1, 2)$ is an EE ordering over K_i.*

The definition of deals remains the same as in [16].

Definition 2. *Let $B = ((K_1, \preceq_1), (K_2, \preceq_2))$ be a bargaining game. A deal of B is a pair (D_1, D_2) satisfying the following two conditions: for each $i = 1, 2$,*

1. *$D_i \subseteq K_i$;*
2. *for each $k = 1, 2, \cdots$, $D_i \cap K_i^k$ is set-inclusion maximal among the subsets of K_i^k such that $\bigcup_{j=1}^{k} (D_i \cap K_i^j) \cup D_{-i}$ is consistent.*

The set of all deals of B is denoted by $\Omega(B)$.

We remark that since K_1 and K_2 are belief sets, it is easy to prove that for any deal (D_1, D_2), both D_1 and D_2 are logically closed. This property will play a key role in the proofs of theorems in Section 4.

Definition 3. *For any bargaining game $B = ((K_1, \preceq_1), (K_2, \preceq_2))$, we call $\Phi = (\Phi_1, \Phi_2)$ the core of the game if*

$$\Phi_1 \stackrel{def}{=} \bigcap_{(D_1,D_2)\in\gamma(B)} D_1, \quad \Phi_2 \stackrel{def}{=} \bigcap_{(D_1,D_2)\in\gamma(B)} D_2$$

where

$\gamma(B) = \{D \in \Omega(B) : \rho_B(D) = \rho_B\}$
$\rho_B(D) \stackrel{def}{=} \min\{\rho_{K_1}(D_1), \rho_{K_2}(D_2)\}$
$\rho_B \stackrel{def}{=} \max\{\rho_B(D) : D \in \Omega(B)\}$

It is easy to see that $\gamma(B)$ represents the subset of $\Omega(B)$ that contains the deals with the highest degree of coverage over all deals in $\Omega(B)$. The min-max construction of the core captures the idea that the final agreement should maximally and evenly satisfy both agents's demands(see [16]).

Now we can finalize the reconstruction of bargaining solution.

Definition 4. *A bargaining solution is a function* **A** *which maps a bargaining game to a set of sentences (agreement), defined as follows. For each bargaining game $B = ((K_1, \preceq_1), (K_2, \preceq_2))$*

$$\mathbf{A}(B) \stackrel{def}{=} (K_1 \otimes_1 \Phi_2) \cap (K_2 \otimes_2 \Phi_1) \tag{1}$$

where (Φ_1, Φ_2) is the core of B and $\otimes_i$ is the prioritized revision function over $(K_i, \preceq_i)$.

We call $\mathbf{A}(B)$ (sometimes we write it as $\mathbf{A}(K_1, K_2)$) *an agreement function.* It is easy to see that the outcomes of an agreement function do not depend on the syntax of its inputs. However, if the bargaining solution defined in [16] takes belief sets as inputs, it will give exactly the same outcomes as the above definition. In such a sense, the logical properties we discuss in the following section can be viewed as the idealized properties of the bargaining solution defined in [16].

4 Logical Properties of Bargaining Solution

In this section, we will present a set of logical properties of the bargaining solution we introduced in the previous section. We will show that the solution satisfies most desired properties for agreement functions and negotiation functions. To establish these properties, we need a few technical lemmas. Note that none of the lemmas holds without the assumption of the logical closeness of belief sets.

Lemma 3. *Given a bargaining game* $B = ((K_1, \preceq_1), (K_2, \preceq_2))$*, let* $\pi_{max} = \max\{k : K_1^{\leq k} \cup K_2^{\leq k}$ *is consistent*$\}$ *and* $(\Psi_1, \Psi_2) = (K_1^{\leq \pi_{max}}, K_2^{\leq \pi_{max}})$*. Then* $(\Phi_1, \Phi_2) = (K_1 \cap (\Psi_1 + \Psi_2), K_2 \cap (\Psi_1 + \Psi_2))$.

Proof. Before we prove the main result of the lemma, we first show that $\rho_B = \pi_{max}$. For any deal $D = (D_1, D_2)$ such that $K_1^{\leq \pi_{max}} \subseteq D_1$ and $K_2^{\leq \pi_{max}} \subseteq D_2$, we have $\rho_B(D) \geq \pi_{max}$. Thus $\rho_B \geq \pi_{max}$. On the other hand, for any $D \in \gamma(B)$, $\rho_B(D) = \rho_B$, which means that $\rho_{K_1}(D_1) \geq \rho_B$ and $\rho_{K_2}(D_2) \geq \rho_B$. According to the definition of degree of coverage of a deal, $K_1^{\leq \rho_{K_1}(D_1)} \subseteq D_1$ and $K_2^{\leq \rho_{K_2}(D_2)} \subseteq D_2$. So either $\rho_{K_1}(D_1) \leq \pi_{max}$ or $\rho_{K_2}(D_2) \leq \pi_{max}$ as $D_1 \cup D_2$ is consistent. Therefore $\rho_B \leq \min\{\rho_{K_1}(D_1), \rho_{K_2}(D_2)\} \leq \pi_{max}$. We have proved that $\rho_B = \pi_{max}$.

Obviously if $K_1 \cup K_2$ is consistent, then $(\Phi_1, \Phi_2) = (K_1, K_2)$. Therefore we will assume that $K_1 \cup K_2$ is inconsistent. We only prove $\Phi_1 = K_1 \cap (\Psi_1 + \Psi_2)$. The second component is similar.

For any $(D_1, D_2) \in \gamma(B)$, we have $\Psi_1 \subseteq D_1$ and $\Psi_2 \subseteq D_2$. In fact, we can prove that $K_1 \cap (\Psi_1 + \Psi_2) \subseteq D_1$. If it is not the case, there exists a sentence $\varphi \in K_1 \cap (\Psi_1 + \Psi_2)$ but $\varphi \notin D_1$. On one hand, $\varphi \in K_1 \cap (\Psi_1 + \Psi_2)$ implies that $D_1 \cup D_2 \vdash \varphi$. On the other hand, $\varphi \notin D_1$ implies that $\{\varphi\} \cup D_1 \cup D_2$ is inconsistent (otherwise D_1 will include φ). It follows that $D_1 \cup D_2 \vdash \neg\varphi$. Therefore $D_1 \cup D_2$ is inconsistent, a contradiction. We have proved that for any deal $(D_1, D_2)\gamma(B)$, $K_1 \cap (\Psi_1 + \Psi_2)$. Thus $K_1 \cap (\Psi_1 + \Psi_2) \subseteq \bigcap_{(D_1,D_2)\in\gamma(B)} D_1 = \Phi_1$.

Now we prove that $\Phi_1 \subseteq K_1 \cap (\Psi_1 + \Psi_2)$. To this end, we assume that $\varphi \in \Phi_1$. If $\varphi \notin \Psi_1 + \Psi_2$, we have $\{\neg\varphi\} \cup \Psi_1 \cup \Psi_2$ is consistent. On the other hand, since $K_1^{\leq \pi_{max}+1} \cup K_2^{\leq \pi_{max}+1}$ is inconsistent, there exists a sentence $\psi \in K_1^{\leq \pi_{max}+1}$ such that $\neg\psi \in K_2^{\leq \pi_{max}+1}$ (because both $K_1^{\leq \pi_{max}+1}$ and $K_2^{\leq \pi_{max}+1}$ are logically closed). Since $\{\neg\varphi\} \cup \Psi_1 \cup \Psi_2$ is consistent, there is a deal $(D_1, D_2) \in \gamma(B)$ such that $\{\neg\varphi \vee \psi\} \cup \Psi_1 \subseteq D_1$ and $\{\neg\varphi \vee \neg\psi\} \cup \Psi_2 \subseteq D_2$. We know that $\varphi \in \Phi_1$, so $\varphi \in D_1 + D_2$. Thus $\psi \wedge \neg\psi \in D_1 + D_2$, a contradiction. ¶

Lemma 4. *Assume that* Ψ_1, Ψ_2 *and* π_{max} *are defined as Lemma 3. Then*

$$K_1 \otimes_1 \Phi_2 = K_1 \otimes_1 \Psi_2 \text{ and } K_2 \otimes_2 \Phi_1 = K_2 \otimes_2 \Psi_1$$

where $\otimes_i$ *is the prioritized revision function over* $(K_i, \preceq_i)$.

Proof. We only present the proof for the first statement. The second one is similar.

First it is easy to prove that $\Psi_1 + \Psi_2 \subseteq K_1 \otimes_1 \Phi_2$. It follows that $K_1 \otimes_1 \Phi_2 = K_1 \otimes_1 \Phi_2 + (\Psi_1 + \Phi_2)$. On the other hand, according to Lemma 3, we have $\Phi_2 \subseteq \Psi_1 + \Psi_2$. Since $\otimes_1$ satisfies the AGM postulates, we then have $K_1 \otimes_1 \Phi_2 + (\Psi_1 + \Phi_2) = K_1 \otimes_1 (\Psi_1 + \Psi_2)$. Therefore $K_1 \otimes_2 \Phi_2 = K_1 \otimes_1 (\Psi_1 + \Psi_2)$. In addition, it is easy to prove that $\Psi_1 \subseteq K_1 \otimes_1 \Psi_2$. By AGM postulates again, we have $K_1 \otimes_1 \Psi_2 = K_1 \otimes_1 \Psi_2 + \Psi_1 = K_1 \otimes_1 (\Psi_1 + \Psi_2)$. Therefore $K_1 \otimes_1 \Phi_2 = K_1 \otimes_1 \Psi_2$.¶

Lemma 5. *Assume that* Ψ_1, Ψ_2 *and* π_{max} *are defined as Lemma 3. Let*

$\Psi_1' = K_1^{\leq \pi^1_{max}}$, *where* $\pi^1_{max} = \max\{k : K_1^{\leq k} \cup K_2^{\leq \pi_{max}}$ *is consistent*$\}$,
$\Psi_2' = K_2^{\leq \pi^2_{max}}$, *where* $\pi^2_{max} = \max\{k : K_1^{\leq \pi_{max}} \cup K_2^{\leq k}$ *is consistent*$\}$.
Then

$$K_1 \otimes_1 \Phi_2 = \Psi_1' + \Psi_2 \text{ and } K_2 \otimes_2 \Phi_1 = \Psi_1 + \Psi_2'.$$

where $\otimes_i$ *is the prioritized selection revision over* $(K_i, \preceq_i)$.

Proof. We only present the proof for the first statement. The second one is similar. According to Lemma 4, $K_1 \otimes_1 \Phi_2 = K_1 \otimes_1 \Psi_2$. Therefore we only need to prove that $K_1 \otimes_1 \Psi_2 = \Psi_1' + \Psi_2$.

If $\Psi_1' = K_1$, $K_1 \cup \Psi_2$ is consistent. Then $K_1 \otimes_1 \Psi_2 = K_1 + \Psi_2$. We have $K_1 \otimes_1 \Psi_2 = K_1 + \Psi_2 = \Psi_1' + \Psi_2$, as desired. If $\Psi_1' \neq K_1$, according to the definition of π^1_{max}, we have $K_1^{\leq \pi^1_{max}+1} \cup K_2^{\leq \pi_{max}}$ is inconsistent. It follows that there exists a sentence $\varphi \in K_1^{\leq \pi^1_{max}+1}$ such that $\neg\varphi \in K_2^{\leq \pi_{max}} = \Psi_2$ (note that both $K_1^{\leq \pi^1_{max}+1}$ and $K_2^{\leq \pi_{max}}$ are logically closed). Now we can come to the conclusion that $\Psi_1' + \Psi_2 = K_1 \otimes_1 \Psi_2$. In fact, by the construction of prioritized belief revision, we can easily verify that $\Psi_1' + \Psi_2 \subseteq K_1 \otimes_1 \Psi_2$. To prove the other direction of inclusion, we assume that $\psi \in K_1 \otimes_1 \Psi_2$. If $\psi \notin \Psi_1' + \Psi_2$, then $\{\neg\varphi\} \cup \Psi_1' \cup \Psi_2$ is consistent. So is $\{\neg\psi \vee \varphi\} \cup \Psi_1' \cup \Psi_2$. Notice that $\neg\psi \vee \varphi \in K_1^{\leq \pi^1_{max}+1}$. There exists $H \in K_1 \Downarrow \Psi_2$ such that $\{\neg\psi \vee \varphi\} \cup \Psi_1' \subseteq H$. Since $\psi \in K_1 \otimes_1 \Psi_2$ and H is logically closed, we have $\varphi \in H$, which contradicts the consistency of $H \cup \Psi_2$. Therefore $K_1 \otimes_1 \Psi_2 \subseteq \Psi_1' + \Psi_2$. ¶

Having the above lemmas, the verification of the following theorems becomes much easier.

The first theorem shows that the calculation of agreement function can be transferred to the calculation of maximal consistent hierarchies of two belief sets.

Theorem 1. *For any bargaining game* $B = ((K_1, \preceq_1), (K_2, \preceq_2))$,

$$\mathbf{A}(B) = (\Psi_1' + \Psi_2) \cap (\Psi_1 + \Psi_2') \tag{2}$$

where Ψ_1, Ψ_1', Ψ_2 *and* Ψ_2' *are defined as Lemma 5.*

Proof. Straightforward from Lemma 5. ¶

We remark that the computation of Ψ_1, Ψ_1', Ψ_2 and Ψ_2' has much less cost than the calculation of the core of a game. This explains why we could reduce the computational complexity of bargaining solution significantly (see Section 5).

The following theorem shows the basic logical properties of agreement function.

Theorem 2. *Let* $\mathbf{A}$ *is a bargaining solution. For any bargaining game* $B = ((K_1, \preceq_1), (K_2, \preceq_2))$*, the following properties hold:*

(A1) $\mathbf{A}(K_1, K_2) = Cn(\mathbf{A}(K_1, K_2))$.
(A2) $\mathbf{A}(K_1, K_2)$ *is consistent.*

(A3) $\mathbf{A}(K_1, K_2) \subseteq K_1 + K_2$.
(A4) *If* $K_1 \cup K_2$ *is consistent, then* $K_1 + K_2 \subseteq \mathbf{A}(K_1, K_2)$.

Given Theorem 1, the verification of the above properties is trivial. One may notice that the above properties are similar to the postulates introduced in [4] for negotiation outcomes. In fact, our agreement function satisfies all the postulates for negotiation outcomes except

(O4) $K_1 \cap K_2 \subseteq A(K_1, K_2)$ or $A(K_1, K_2) \cup (K_1 \cap K_2) \models \bot$.

The following is a counterexample.

Example 1. *Let* $K_1 = Cn(\{p, \neg q, r\})$ *with the hierarchy* $K_1^1 = Cn(\{p\})$ *and* $K_1^2 = K_1 \backslash K_1^1$. *Let* $K_2 = Cn(\{q, \neg p, r\})$ *with the hierarchy* $K_2^1 = Cn(\{q\})$ *and* $K_2^2 = K_2 \backslash K_2^1$. *Then* $\Psi_1 = \Psi_1' = Cn(\{p\})$ *and* $\Psi_2 = \Psi_2' = Cn(\{q\})$. *It follows that* $A(K_1, K_2) = Cn(\{p, q\})$. *Obviously* **(O4)** *does not hold for this example.*

The reason that both agents give up their common demand r is the following. If an agent demands $\neg p$ or $\neg q$, the agent needs to commit herself to accept its logical consequence $\neg p \vee \neg r$ or $\neg q \vee \neg r$, respectively. Since r is less entrenched than the commitment by both agents, they have to give up r in order to reach the agreement $Cn(\{p, q\})$. If the agents do not mean that, it should be explicitly expressed in the initial demands.

The following theorem shows that with our construction of bargaining solution, we can define a negotiation function which is similar to the negotiation function introduced by Zhang *et al.* in [14].

Theorem 3. Let N be a function defined as follows:

$$N(K_1, K_2) = (K_1 \otimes_1 \Phi_2, K_2 \otimes_2 \Phi_1)$$

where (Φ_1, Φ_2) is the core of the bargaining game $B = ((K_1, \preceq_1), (K_2, \preceq_2))$. Then N has the following properties (N_i is the i-th component of N):

(N1) $N_i(K_1, K_2) = Cn(N_i(K_1, K_2))$. (*Closure*)
(N2) $N_i(K_1, K_2) \subseteq K_1 + K_2$. (*Inclusion*)
(N3) If $K_1 \cup K_2$ is consistent, then $K_1 + K_2 \subseteq N_i(K_1, K_2)$. (*Vacuity*)
(N4) If $K_i \cup N_i(K_1, K_2)$ is consistent, then $K_i \subseteq N_i(K_1, K_2)$. (*Consistent Expansion*)
(N5) If $K_i \cup N_{-i}(K_1, K_2)$ is consistent, then $N_{-i}(K_1, K_2) \subseteq N_i(K_1, K_2)$. (*Safe Expansion*)

Proof. The proof of (N1)-(N4) is trivial by Theorem 1. For (N5), since $K_i \cup (\Psi'_{-i} + \Psi_i)$ is consistent, we have $\pi_{max}^{-i} = \pi_{max}$. It follows that $N_{-i}(K_1, K_2) = \Psi_1 + \Psi_2$. Therefore $N_{-i}(K_1, K_2) \subseteq N_i(K_1, K_2)$. ¶

We can see that the properties of *Closure, Inclusion, Vacuity and Consistent* are exactly the same as the corresponding postulates in [14]. The postulate *Extensionality* is trivially true for our definition. The postulates *Inconsistency and Iteration* are not applicable in our case because we do not consider inconsistent inputs and iteration operations. The following example illustrates that the postulate *No Recantation* is invalid for our definition of negotiation function.

Example 2. *Let* $K_1 = Cn(\{p, \neg q, r\})$ *with the hierarchy* $K_1^1 = Cn(\{p\})$, $K_1^2 = Cn(\{p, \neg q\}) \backslash K_1^1$ *and* $K_1^3 = K_1 \backslash K_1^{\leq 2}$. *Let* $K_2 = Cn(\{q, r\})$ *with the hierarchy* $K_2^1 = Cn(\{q\})$ *and* $K_2^2 = K_2 \backslash K_2^1$. *Then* $\Psi_1 = \Psi_1' = Cn(\{p\})$, $\Psi_2 = Cn(\{q\})$ *and* $\Psi_2' = Cn(\{q, r\})$. *It follows that* $N_1(K_1, K_2) = Cn(\{p, q\})$ *and* $N_2(K_1, K_2) = Cn(\{p, q, r\})$. *Therefore* $r \in K_1 \cap N_2(K_1, K_2)$ *but* $r \notin N_1(K_1, K_2)$.

As a weak version of *No Recantation* the property of *Safe Expansion* says that if an agent initiatively gives up all the demands which conflicts to the other agent, the other agent should accept all the consistent demands from the first agent.

5 Computational Complexity

In [16], Zhang and Zhang shows that the complexity of belief-revision-based bargaining solutions is Π_2^P-hard. This result is based on the syntax-dependent construction of bargaining solution.

Theorem 4. [Zhang and Zhang 2006] *Let* B *be a bargaining game and* φ *a formula. Deciding whether* $\mathbf{A}(B) \vdash \varphi$ *is* Π_2^P*-hard, where* $\mathbf{A}(B)$ *is the agreement function defined in [16].*

In this section, we will show that the complexity can be reduced if we use the syntax-independent construction of agreement function.

We assume that readers are familiar with the complexity classes of P, NP, coNP, $\Delta_2^P[\mathcal{O}(\log n)]$, Δ_2^P and Π_2^P. It is well known that $P \subseteq NP \subseteq \Delta_2^P[\mathcal{O}(\log n)] \subseteq \Delta_2^P \subseteq \Pi_2^P$, and these inclusions are generally believed to be proper (readers may refer to [10] for further details).

Given a bargaining game $B = ((K_1, \preceq_1), (K_2, \preceq_2))$, since K_1 and K_2 are logically closed, they are infinite sets even though the language we consider is finite. To make computation possible, we assume that equivalent statements are represented by only one sentence, so a belief set can be finite[3]. In such a sense, we will refer a bargaining game B to a pair of prioritized belief sets, $((X_1, \preceq_1), (X_2, \preceq_2))$, where X_i is finite sets of sentences and $\preceq_i$ is a pre-order over X_i which satisfies logic constraint (LC). According to Lemma 1, for each $i = 1, 2$, we can always write $X_i = X_i^1 \cup \cdots \cup X_i^m$, where $X_i^k \cap X_i^l = \emptyset$ for any $k \neq l$. Also for each $k < m$, if a formula $\varphi \in X_i^k$, then there does not exist a $\psi \in X_i^l$ $(k < l)$ such that $\varphi \prec_i \psi$. Therefore, for the convenience of our complexity analysis, in the rest of this section, we will specify a bargaining game as $B = (X_1, X_2)$, where $X_1 = \bigcup_{i=1}^m X_1^i$ and $X_2 = \bigcup_{j=1}^n X_2^j$, and $X_1^1, \cdots, X_1^m$, and $X_2^1, \cdots, X_2^n$ are the partitions of X_1 and X_2 respectively and satisfy the property mentioned above. According to Theorem 1, we can define an agreement function by Equation (2) as

$$\mathbf{A}(B) = (\Psi_1' + \Psi_2) \cap (\Psi_1 + \Psi_2') \tag{3}$$

[3] In fact, the complexity results presented in this section do not require the belief sets in a bargaining game to be logically closed. We can view Equation (2) as the approximation of bargaining solution.

where

$(\Psi_1, \Psi_2) = (\bigcup_{i=1}^{\pi} X_1^i, \bigcup_{i=1}^{\pi} X_2^i)$;

$(\Psi_1', \Psi_2') = (\bigcup_{i=1}^{\pi_1} X_1^i, \bigcup_{i=1}^{\pi_2} X_2^i)$;

$\pi = \max\{k : (\bigcup_{i=1}^{k} X_1^i) \cup (\bigcup_{i=1}^{k} X_2^i)$ is consistent$\}$;

$\pi_1 = \max\{k : (\bigcup_{i=1}^{k} X_1^i) \cup (\bigcup_{i=1}^{\pi} X_2^i)$ is consistent$\}$;

$\pi_2 = \max\{k : (\bigcup_{i=1}^{\pi} X_1^i) \cup (\bigcup_{i=1}^{k} X_2^i)$ is consistent$\}$.

Theorem 5. *Let $B = (X_1, X_2)$ be a bargaining game and φ a formula. Deciding whether $\mathbf{A}(B) \vdash \varphi$ is $\Delta_2^P[\mathcal{O}(\log n)]$-complete.*

Proof. Membership proof. Let $B = (X_1, X_2)$, where $X_1 = \bigcup_{i=1}^{m} X_1^i$ and $X_2 = \bigcup_{i=1}^{n} X_2^j$. Firstly, we can find a maximal k with a binary search such that if $\bigcup_{i=1}^{k} X_1^k \cup \bigcup_{i=1}^{k} X_2^k$ is consistent but $\bigcup_{i=1}^{k+1} X_1^i \cup \bigcup_{i=1}^{k+1} X_2^i$ is no longer consistent. Obviously, we will need $\mathcal{O}(\log n)$ times search. Secondly, we fix $\bigcup_{i=1}^{k} X_1^i$, and find a maximal p such that $\bigcup_{i=1}^{k} X_1^i \cup \bigcup_{j=1}^{p} X_2^j$ is consistent but $\bigcup_{i=1}^{k} X_1^k \cup \bigcup_{j=1}^{p+1} X_2^j$ is not inconsistent. Note that we should have $k \leq p$. Also, this can be done with a binary search in time $\mathcal{O}(\log n)$. In a similar way, we can fix $\bigcup_{i=1}^{k} X_2^i$ and find a maximal q satisfying that $\bigcup_{i=1}^{q} X_1^i \cup \bigcup_{j=1}^{k} X_2^j$ is consistent and $\bigcup_{i=1}^{q+1} X_1^i \cup \bigcup_{j=1}^{k} X_2^i$ is not consistent. So we can compute $\mathbf{A}(B)$ using a deterministic Turing machine with $\mathcal{O}(\log n)$ queries to an NP oracle. Finally, we check the consistency of $\mathbf{A}(B) \cup \{\neg\varphi\}$ with one query to an NP oracle. So the problem is in $\Delta_2^P[\mathcal{O}(\log n)]$.

Hardness proof. By restrict $B = (X_1, X_2)$ where $X_1 = \{\varphi_1\} \cup \cdots \cup \{\varphi_n\}$ and $X_2 = \{\psi\}$. Then it is easy to see that our bargaining problem is identical to the cut base revision, which implies that deciding whether $\mathcal{A}(B) \vdash \varphi$ is $\Delta_2^P[\mathcal{O}(\log n)]$-hard [7]. ¶

The following result shows that if we restricts the language to be Horn clauses, the decision problem of bargaining solution is tractable.

Theorem 6. *Let B be a bargaining game and φ a formula where all formulas occurring in B and φ are Horn clauses. Deciding whether $\mathbf{A}(B) \vdash \varphi$ is in P.*

6 Conclusion and Related Work

In this paper, we have presented a set of logical properties of belief-revision-based bargaining solution. By representing bargaining game as a pair of prioritized belief sets, the computation of bargaining solution can be converted to the construction of maximal consistent hierarchy of two agents' belief sets. Based on the result, we have shown that the agreement function and negotiation function defined by the bargaining solution satisfies most of postulates introduced in the literature. Our complexity analysis indicates that in general the computation of belief-revision-based bargaining solution can be reduced or be approximated to $\Delta_2^P[\mathcal{O}(\log n)]$-complete.

This work is closed related to [16]. In fact, we can view the syntax-independent bargaining solution is a special case of syntax-dependent bargaining solution when bargaining games consists of belief sets. Although the assumption of logical closeness is just an idealized case, it is essential to disclose the logical properties behind the negotiation reasoning. We have shown that our solution to negotiation problem is different from the axiomatic approaches [4,5,14]. Since these formalisms are all based on the assumption of logical closeness, it is possible to apply our approach to develop a concrete construction for their negotiation functions.

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