A Fixed-Point Property of Logic-Based Bargaining Solution

Dongmo Zhang

Intelligent Systems Laboratory
University of Western Sydney, Australia
dongmo@scm.uws.edu.au

Abstract. This paper presents a logic-based bargaining solution based on Zhang and Zhang's framework. It is shown that if the demand sets of players are logically closed, the solution satisfies a fixed-point property, which says that the outcome of bargaining is the result of mutual belief revision. The result is interesting not only because it presents a desirable logical property of bargaining solution but also establishes a link between bargaining theory and multi-agent belief revision.

1 Introduction

Negotiation or bargaining is a process of dispute resolution to reach mutually beneficial agreements. The studies of negotiation in game theory, known as *bargaining theory*, initiated by John Nash's path-breaking work [1], has reached a high sophistication with a variety of models and solutions and has been extensively applied to economics, sociology, management science, and politics [2,3,4].

The game-theoretical model of bargaining is purely numerical. Although the numerical theory of bargaining provides "a 'clear-cut' numerical predication for a wide range of bargaining problems", it does not help us to understand how disputes are resolved through a bargaining process ([5] p.81-88).

In recent years, the AI researchers try to rebuild the theory of bargaining and negotiation in order to model logical reasoning behind a bargaining process. Kraus *et al.* introduced a logical model of negotiation based on argumentation theory [6,7]. Unlike game theory, the model allows explicit representation of negotiation items, promises, threats and arguments. More importantly, bargaining process can be embedded into logic-based multi-agent systems so that negotiation becomes a component of agent planning. Similar to Rubinstein's strategic model of bargaining, the argumentation-based approach views bargaining as a non-cooperative game. Zhang *et al.* introduced a logical model of negotiation based on belief revision theory [8,9,10]. Different from the argumentation-based framework, the belief-revision-based approach takes a cooperative view. In order to reach an agreement, each player tries to persuade the other player to accept her demands or beliefs. Anyone who is convinced to accept the other player's demands will need to conduct a course of belief revision. It was assumed that

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any possible outcome of negotiation, (Ψ_1, Ψ_2), should satisfies the following fixed-point condition [11], which says that the outcome of negotiation is the common demands or beliefs after mutual belief revision:

$$Cn(\Psi_1 \cup \Psi_2) = (Cn(X_1) \otimes_1 \Psi_2) \cap (Cn(X_2) \otimes_2 \Psi_1)$$

where X_i contains the demands of agent i and $\otimes_i$ is the belief revision operator of agent i. However, there is no justification for the assumption. This paper aims to build a concrete bargaining solution to satisfy the fixed-point condition. The construction of the solution is based on the bargaining model proposed by Zhang and Zhang in [12,13]. The result of the paper not only shows the logical property of bargaining but also establishes the link between bargaining and belief revision, which may be helpful for the investigation of multi-agent belief revision.

2 Logical Model of Bargaining

Within this paper, we consider the bargaining situations with two players. We assume that each party has a set of negotiation items, referred to as *demand set*, described by a finite propositional language $\mathcal{L}$. The language is that of classical propositional logic with an associated consequence operation Cn in the sense that $Cn(X) = \{\varphi : X \vdash \varphi\}$, where X is a set of sentences. A set X of sentences is *logically closed* or called a *belief set* when $X = Cn(X)$. If X and Y are two sets of sentences, $X + Y$ denotes $Cn(X \cup Y)$.

Suppose that X_1 and X_2 are the demand sets from two bargaining parties respectively. To simplify exploration, we use X_{-i} to represent the other set among X_1 and X_2 if X_i is one of them. If D is a vector of two components, D_1 and D_2 will represent each of the components of D.

2.1 Bargaining Games

We will use the bargaining model introduced by Zhang and Zhang in [12] to represent a bargaining situation.

Definition 1. [12] *A bargaining game is a pair* $((X_1, \succeq_1), (X_2, \succeq_2))$, *where* X_i $(i = 1, 2)$ *is a logically consistent set of sentences in* $\mathcal{L}$ *and* $\succeq_i$ *is a complete transitive reflexive order (total preorder or weak order) over* X_i *which satisfies the following logical constraints*[1]*:*

(LC) *If* $\varphi_1, \cdots, \varphi_n \vdash \psi$, *then there is* k $(1 \leq k \leq n)$ *such that* $\psi \succeq_i \varphi_k$.

[1] A complete transitive reflexive order, i.e., total preorder or weak order, satisfies the following properties:
- Completeness or totality: $\varphi \preceq \psi$ or $\psi \preceq \varphi$.
- Reflexivity: $\varphi \preceq \varphi$.
- Transitivity: if $\varphi \preceq \psi$ and $\psi \preceq \chi$ then $\varphi \preceq \chi$.

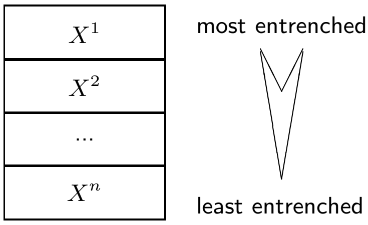

Fig. 1. The hierarchy of a demand set

We call the pair $(X_i, \succeq_i)$ *the prioritized demand set of player* i*. For any* $\varphi, \psi \in X_i$*,* $\psi \succ \varphi$ *denotes that* $\psi \succeq_i \varphi$ *and* $\varphi \not\succeq_i \psi$*.* $\psi \approx_i \varphi$ *denotes that* $\psi \succeq_i \varphi$ *and* $\varphi \succeq_i \psi$*.*

Intuitively, a bargaining game is a formal representation of a bargaining situation whereby each player describes his demands in logical formulae and expresses his preferences over his demands in total preorder. We assume that each player has consistent demands. The preference ordering of each player reflects the *degree of entrenchment* in which the player defends his demands. The logical constraint (LC) says that if $\varphi_1, \cdots, \varphi_n$ and ψ are all your demands and $\varphi_1, \cdots, \varphi_n \vdash \psi$, then ψ should not be less entrenched than all the φ_i because if you fail to defend ψ, at least one of the φ_i has to be dropped (otherwise you would not have lost ψ). This indicates that the preference orderings are different from players' payoff or utility. For instance, suppose that p_1 represents the demand of a seller "*the price of the good is no less than \$10*" and p_2 denotes "*the price of the good is no less than \$8*". Obviously the seller could get higher payoff from p_1 than p_2. However, since p_1 implies p_2, she will entrench p_2 no less than p_1, *i.e.*, $p_2 \succeq p_1$, because, if she fails to keep p_1, she can still bargain for p_2 but the loss of p_2 means the loss of both.

Given a prioritized demand set $(X, \succeq)$, we define recursively a hierarchy, $\{X^j\}_{j=1}^{+\infty}$, of X with respect to the ordering $\succeq$ as follows:

1. $X^1 = \{\varphi \in X : \neg\exists\psi \in X(\psi \succ \varphi)\}$; $T^1 = X \backslash X^1$.
2. $X^{j+1} = \{\varphi \in T^j : \neg\exists\psi \in T^j(\psi \succ \varphi)\}$; $T^{j+1} = T^j \backslash X^{j+1}$.

where $\psi \succ \varphi$ denotes $\psi \succeq \varphi$ and $\varphi \not\succeq \psi$. The intuition behind the construction is that, at each stage of the construction, we collects all maximal elements from the current demand set and remove them from the set for the next stage of the construction. It is easy to see that there exists a number n such that $X = \bigcup_{j=1}^{n} X^j$ due to the logical constraint LC[2].

It is easy to see that for any $\varphi \in X^j$ and $\psi \in X^k$, $\varphi \succ \psi$ if and only if $j < k$. In the sequel, we write $X^{\leq k}$ to denote $\bigcup_{j=1}^{k} X^j$.

[2] Note that X can be an infinite set even though the language is finite.

Based on the hierarchy of each demand, we can define a belief revision function for each agent by following Nebel's idea of *prioritized base revision* [14]:

For any demand set $(X, \preceq)$ and a set, F, of sentences,

$$X \otimes F \overset{def}{=} \bigcap_{H \in X \Downarrow F} (H + F),$$

where $X \Downarrow F$ is defined as: $H \in X \Downarrow F$ if and only if

1. $H \subseteq X$,
2. for all k $(k = 1, 2, \cdots)$, $H \cap X^k$ is a maximal subset of X^k such that $\bigcup_{j=1}^{k} (H \cap X^j) \cup F$ is consistent.

In other words, H is a maximal subset of X that is consistent with F and gives priority to the higher ranked items. The following result will be used in Section 3.

Lemma 1. [14] *If X is logically closed, then $\otimes$ satisfies all AGM postulates.*

2.2 Possible Agreements

Similar to [12], we define a possible outcome of negotiation as a concession made by two players.

Definition 2. *Let $G = ((X_1, \succeq_1), (X_2, \succeq_2))$ be a bargaining game. A deal of G is a pair (D_1, D_2) satisfying the following conditions: for each $i = 1, 2$,*

1. *$D_i \subseteq X_i$;*
2. *$X_1 \cap X_2 \subseteq D_i$;*
3. *for each k $(k = 1, 2, \cdots)$, $D_i \cap X_i^k$ is a maximal subset of X_i^k such that $\bigcup_{j=1}^{k} (D_i \cap X_i^j) \cup D_{-i}$ is consistent.*

where $\{X_i^j\}_{k=1}^{+\infty}$ is the hierarchy of X_i. The set of all deals of G is denoted by $\Omega(G)$, called the feasible set of the game.

Intuitively, a possible agreement is a pair of subsets of two players' original demand sets such that the collection of remaining demands is consistent. Obviously each player would like to keep as many original demands as possible. Therefore, if a player has to give up a demand, the player typically gives up the ones with the lowest priority. Note that we require that no player gives up common demands, which is crucial to the fixed-point property. This is different from Zhang and Zhang's definition in [12].

2.3 Bargaining Solution

We have shown how to generates all possible deals from a bargaining game. However, a game might have multiple deals. Different deals would be in favor of

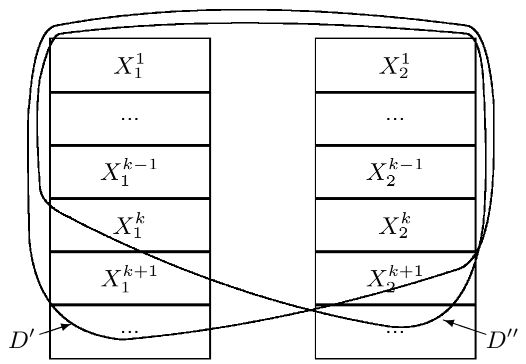

Fig. 2. Different deals are in favour of different parties

different parties. The major concern of a bargaining theory is how to measure and balance the gain of each negotiating party.

Instead of counting the number of demands a deal contains for each party, we consider the top block demands a player keeps in the deal (the top levels of demands in each player's demand hierarchy) and ignore all demands that are not included in the top blocks except the common demands[3].

Given a deal D, we shall use the maximal top levels of each player's demands the deal contains as the indicator of the player's gain from the deal, i.e., $\max\{k : X_i^{\leq k} \subseteq D_i\}$. For instance, in Figure 2, player 1 can successfully remain maximally top $k+1$ levels of his demands from deal D' while player 2 gains maximally top k levels of his demands from the deal.

To compare players' gains from different deals, we use the gain of the player with smaller gain from a deal as the index of the deal, i.e., $\min\{\max\{k : X_1^{\leq k} \subseteq D_1\}, \max\{k : X_2^{\leq k} \subseteq D_2\}\}$, or equivalently, $\max\{k : X_1^{\leq k} \subseteq D_1 \text{ and } X_2^{\leq k} \subseteq D_2\}$. For instance, in Figure 2, the gain index of D' is k while the gain index of D'' is $k-1$. By using this index, we can collect all the best deals of a game:

$$\gamma(G) = \arg \max_{(D_1,D_2)\in\Omega(G)} \{k : X_1^{\leq k} \subseteq D_1 \text{ and } X_2^{\leq k} \subseteq D_2\}$$

Based on the intuitive description, we are now ready to construct our bargaining solution.

Definition 3. *A bargaining solution is a function F which maps a bargaining game $G = ((X_1, \succeq_1), (X_2, \succeq_2))$ to a pair of sets of sentences defined as follows:*

$$F(G) \stackrel{def}{=} \Big(\bigcap_{(D_1,D_2)\in\gamma(G)} D_1, \bigcap_{(D_1,D_2)\in\gamma(G)} D_2\Big) \quad (1)$$

where $\gamma(G) = \arg \max_{(D_1,D_2)\in\Omega(G)} \{k : X_1^{\leq k} \subseteq D_1$ *and* $X_2^{\leq k} \subseteq D_2\}$.

[3] Note that common demands of two parties are always included in a deal no matter how much priorities they have.

Let

$$\pi^G_{max} = \max_{(D_1,D_2)\in\Omega(G)} \{k : X_1^{\leq k} \subseteq D_1 \text{ and } X_2^{\leq k} \subseteq D_2\} \quad (2)$$

and

$$(\Phi_1, \Phi_2) = (X_1^{\leq \pi^G_{max}}, X_2^{\leq \pi^G_{max}}) \quad (3)$$

We call $\Phi = (\Phi_1, \Phi_2)$ the *core of the game*. Intuitively, the core of the game is the pair of maximal top block demands that are contained in all the best deals.

To help the reader to understand our solution, let us consider the following example.

Example 1. *A couple are making their family budget for the next year. The husband wants to change his car to a new fancy model and have a domestic holiday. The wife is going to implement her dream of a romantic trip to Europe and suggests to redecorate the kitchen. Both of them know that they can't have two holidays in one year. They also realize that they cannot afford a new car and an overseas holiday in the same year without getting a loan from the bank. However, the wife does not like the idea of borrowing money.*

In order to represent the situation in logic, let c denote "buy a new car", d stand for "domestic holiday", o for "overseas holiday", k for "kitchen redecoration" and l for "loan". Then $\neg(d \wedge o)$ means that it is impossible to have both domestic holiday and overseas holiday. The statement $(c \wedge o) \to l$ says that if they want to buy a new car and also have an overseas holiday, they have to get a loan from the bank.

With the above symbolization, we can express the husband's demands in the following set:

$$X_1 = \{c, d, \neg(d \wedge o), (c \wedge o) \to l\}$$

Similarly, the wife's demands can be represented by:

$$X_2 = \{o, k, \neg(d \wedge o), (c \wedge o) \to l, \neg l\}$$

Assume that the husband's preferences over his demands are:

$$\neg(d \wedge o) \approx_1 (c \wedge o) \to l \succ_1 c \succ_1 d$$

and the wife's preferences are:

$$\neg(d \wedge o) \approx_2 (c \wedge o) \to l \succ_2 o \succ_2 k \succ_2 \neg l$$

Let G represent the bargaining game. It is easy to calculate that the game has the following three possible deals:

$D^1 = (\{\neg(d \wedge o), (c \wedge o) \to l, c, d\}, \{\neg(d \wedge o), (c \wedge o) \to l, k, \neg l\})$.

$D^2 = (\{\neg(d \wedge o), (c \wedge o) \to l, c\}, \{\neg(d \wedge o), (c \wedge o) \to l, o, k\})$.

$D^3 = (\{\neg(d \wedge o), (c \wedge o) \to l\}, \{\neg(d \wedge o), (c \wedge o) \to l, o, k, \neg l\})$.

The core of the game is then:

$$(\{\neg(d \wedge o), (c \wedge o) \to l, c\}, \{\neg(d \wedge o), (c \wedge o) \to l, o\})$$

$\gamma(G)$ contains only a single deal, which is D^2. The solution is then

$$F(G) = D^2 = (\{\neg(d \wedge o), (c \wedge o) \rightarrow l, c\}, \{\neg(d \wedge o), (c \wedge o) \rightarrow l, o, k\})$$

In words, the couple agree upon the commonsense that they can only have one holiday and they have to get a loan if they want to buy a new car and to go overseas for holiday. The husband accepts his wife's suggestion to have holiday in Europe and the wife agrees on buying a new car. As a consequence of the agreement, they agree on getting a loan to buy the car.

3 Fixed Point Property

In [11], it was argued that a procedure of negotiation can be viewed as a course of mutual belief revision when players' belief states with respect to the negotiation are specified by the demand sets of the bargaining game.

Before we show the fixed-point property of the solution we construct, let us consider two facts on the solution:

Lemma 2. $\pi^G_{max} = \max\{k : X_1^{\leq k} \cup X_2^{\leq k} \cup (X_1 \cap X_2)$ *is consistent*$\}$.

Proof. Let $\pi = \max\{k : X_1^{\leq k} \cup X_2^{\leq k} \cup (X_1 \cap X_2)$ is consistent$\}$. It is easy to show that $X_1^{\leq \pi^G_{max}} \cup X_2^{\leq \pi^G_{max}} \cup (X_1 \cap X_2)$ is consistent because $\gamma(G)$ is non-empty. Therefore $\pi^G_{max} \leq \pi$. On the other hand, since $X_1^{\leq \pi} \cup X_2^{\leq \pi} \cup (X_1 \cap X_2)$ is consistent, there exists a deal $(D_1, D_2) \in \Omega(G)$ such that $X_i^{\leq \pi} \subseteq D_i$ and $X_1 \cap X_2 \subseteq D_i$ for each $i = 1, 2$. Thus $\pi \leq \pi^G_{max}$. We conclude that $\pi = \pi^G_{max}$.

Lemma 3. *Given a bargaining game G, for any deal $D \in \Omega(G)$,*

$$D \in \gamma(G) \textit{ iff } \Phi_1 \subseteq D_1 \textit{ and } \Phi_2 \subseteq D_2.$$

where (Φ_1, Φ_2) is the core of G.

Proof. "⇒" Straightforward from the definition of $\gamma(G)$.

"⇐" For any deal $D \in \Omega(G)$, if $\Phi_1 \subseteq D_1$ and $\Phi_2 \subseteq D_2$, then for each i, $X_i^{\leq \pi^G_{max}} \subseteq D_i$. It follows that $\max\{k : X_1^{\leq k} \subseteq D_1$ and $X_2^{\leq k} \subseteq D_2\} \geq \pi^G_{max}$. Therefore $\max\{k : X_1^{\leq k} \subseteq D_1$ and $X_2^{\leq k} \subseteq D_2\} = \pi^G_{max}$.

The above results show an intuitive procedure to construct a bargaining solution. First calculate the core by going through both parties's hierarchies of demands in parallel top-down to the level at which the collective demands are maximally consistent with the common demands. Then collect all the deals that contain the core. Finally, calculate the intersection of the deals that contain the core for each party.

Assume that X_1 and X_2 are two belief sets (so logically closed), representing the belief states of two agents. Mutual belief revision between the agents means that each agent takes part of the other agent's beliefs to revise his belief set. For instance, if Ψ_1 is a subset of X_1 and Ψ_2 is a subset of X_2, then $X_1 \otimes_1 \Psi_2$ is the revised belief set of player 1 after he accepts player 2's beliefs Ψ_2 while $X_2 \otimes_2 \Psi_1$ is the resulting belief set of player 2 after accepting Ψ_1. Such an interaction of belief revision can continue until it reaches a fixed point where the beliefs in common, $(X_1 \otimes_1 \Psi_2) \cap (X_2 \otimes_2 \Psi_1)$, are exactly the beliefs that the agents mutually accept, $\Psi_1 + \Psi_2$. This gives

$$\Psi_1 + \Psi_2 = (X_1 \otimes_1 \Psi_2) \cap (X_2 \otimes_2 \Psi_1) \tag{4}$$

Suppose that the belief sets, X_1 and X_2, represent the two agents' demands, respectively. Then $(X_1 \otimes_1 \Psi_2) \cap (X_2 \otimes_2 \Psi_1)$ should represent the common revised demands after negotiation if Ψ_1 and Ψ_2 are the agreements that are mutually accepted each other. Therefore any bargaining solution should satisfy the fixed-point condition (4). The following theorem confirms that the solution we constructed in this paper satisfies the fixed-point condition.

Theorem 1. *For any bargaining game $G = ((X_1, \succeq_1), (X_2, \succeq_2))$, if X_1 and X_2 are logically closed, the bargaining solution $F(G)$ satisfies the following fixed-point condition:*

$$F_1(G) + F_2(G) = (X_1 \otimes_1 F_2(G)) \cap (X_2 \otimes_2 F_1(G)) \tag{5}$$

where $\otimes_i$ is the prioritized revision operator for player i.

To show this theorem, we need a few technical lemmas.

Lemma 4. *For any bargaining game $G = ((X_1, \succeq_1), (X_2, \succeq_2))$,*

1. *$F_1(G) \subseteq X_1 \otimes_1 F_2(G)$;*
2. *$F_2(G) \subseteq X_2 \otimes_2 F_1(G)$.*

Proof. According to the definition of prioritized base revision, we have $X_1 \otimes_1 F_2(G) = \bigcap_{H \in X_1 \Downarrow F_2(G)} Cn(H \cup F_2(G))$. For any $H \in X_1 \Downarrow F_2(G)$, there is a deal $(D_1, D_2) \in \Omega(G)$ such that $D_1 = H$. This is because we can extend the pair $(H, F_2(G))$ to a deal (H, D_2) such that $F_2(G) \subseteq D_2$. On the other hand, since $\Phi_1 \cup F_2(G)$ is consistent, we have $\Phi_1 \subseteq H$, where (Φ_1, Φ_2) is the core of G. Thus, $\Phi_1 \subseteq D_1$ and $\Phi_2 \subseteq D_2$. According to Lemma 3, we have $(D_1, D_2) \in \gamma(G)$. Since $F_1(G) \subseteq D_1$, we have $F_1(G) \subseteq H$. We conclude that $F_1(G) \subseteq X_1 \otimes_1 F_2(G)$. The proof of the second statement is similar.

By this lemma we have,

1. $F_1(G) + F_2(G) \subseteq X_1 \otimes_1 F_2(G)$;
2. $F_1(G) + F_2(G) \subseteq X_2 \otimes_2 F_1(G)$.

Note that the above lemma does not require the demand sets X_1 and X_2 to be logically closed. However, the following lemmas do.

Lemma 5. *Let (Φ_1, Φ_2) be the core of game $G = ((X_1, \succeq_1), (X_2, \succeq_2))$. If X_1 and X_2 are logically closed, then*

1. $X_1 \otimes_1 F_2(G) = X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$;
2. $X_2 \otimes_2 F_1(G) = X_2 \otimes_2 (\Phi_1 + (X_1 \cap X_2))$

Proof. We only present the proof of the first statement. The second one is similar. Firstly, we prove that $F_2(G) \subseteq \Phi_1 + \Phi_2 + (X_1 \cap X_2)$. If $X_1 \cup X_2$ is consistent, the result is obviously true. Therefore we can assume that $X_1 \cup X_2$ is inconsistent.

Assume that $\varphi \in F_2(G)$. If $\varphi \notin \Phi_1 + \Phi_2 + (X_1 \cap X_2)$, we have $\{\neg\varphi\} \cup \Phi_1 \cup \Phi_2 \cup (X_1 \cap X_2)$ is consistent. According to Lemma 2, we have $X_1^{\leq \pi^G_{max}+1} \cup X_2^{\leq \pi^G_{max}+1} \cup (X_1 \cap X_2)$ is inconsistent. Since our language is finite and both X_1 and X_2 are logically closed, the sets $X_1 \cap X_2$, $X_1^{\leq \pi^G_{max}+1}$ and $X_2^{\leq \pi^G_{max}+1}$ are all logically closed (the latter two due to LC). Therefore each set has a finite axiomatization. Let sentence ψ_0 axiomatize $X_1 \cap X_2$, ψ_1 axiomatize $X_1^{\leq \pi^G_{max}+1}$ and ψ_2 axiomatize $X_2^{\leq \pi^G_{max}+1}$. Thus $\psi_0 \wedge \psi_1 \wedge \psi_2$ is inconsistent. Notice that $\psi_0 \wedge \psi_1 \in X_1$ and $\psi_0 \wedge \psi_2 \in X_2$. It follows that $\neg\varphi \vee (\psi_0 \wedge \psi_1) \in X_1$ and $\neg\varphi \vee (\psi_0 \wedge \psi_2) \in X_2$. Since $\{\neg\varphi\} \cup \Phi_1 \cup \Phi_2 \cup (X_1 \cap X_2)$ is consistent, there is a deal $(D_1, D_2) \in \gamma(G)$ such that $\{\neg\varphi \vee (\psi_0 \wedge \psi_1)\} \cup \Phi_1 \cup (X_1 \cap X_2) \subseteq D_1$ and $\{\neg\varphi \vee (\psi_0 \wedge \psi_2)\} \cup \Phi_2 \cup (X_1 \cap X_2) \subseteq D_2$. We know that $\varphi \in F_2(G)$, so $\varphi \in D_1 + D_2$. Thus $\psi_0 \wedge \psi_1 \wedge \psi_2 \in D_1 + D_2$, which contradicts the fact that $D_1 + D_2$ is consistent. Therefore, we have shown that $F_2(G) \subseteq \Phi_1 + \Phi_2 + (X_1 \cap X_2)$.

Now we prove that $X_1 \otimes_1 F_2(G) = X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$. By Lemma 4, we have $\Phi_1 + \Phi_2 \subseteq X_1 \otimes_1 F_2(G)$. It follows that $X_1 \otimes_1 F_2(G) = (X_1 \otimes_1 F_2(G)) + (\Phi_1 + \Phi_2)$. Furthermore, we yield $X_1 \otimes_1 F_2(G) = (X_1 \otimes_1 F_2(G)) + (\Phi_1 + \Phi_2) + (X_1 \cap X_2)$ because $X_1 \cap X_2 \subseteq F_2(G)$. Since $F_2(G) \subseteq \Phi_1 + \Phi_2 + (X_1 \cap X_2)$. According to the AGM postulates, we have $(X_1 \otimes_1 F_2(G)) + (\Phi_1 + \Phi_2 + (X_1 \cap X_2)) = X_1 \otimes_1 (\Phi_1 + \Phi_2 + (X_1 \cap X_2))$. Therefore $X_1 \otimes_2 F_2(G) = X_1 \otimes_1 (\Phi_1 + \Phi_2 + (X_1 \cap X_2))$. In addition, it is easy to prove that $\Phi_1 \subseteq X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$. By the AGM postulates again, we have $X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) = (X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) + \Phi_1 = X_1 \otimes_1 (\Phi_1 + \Phi_2 + (X_1 \cap X_2))$. Therefore $X_1 \otimes_1 F_2(G) = X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$.

The following lemma will complete the proof of Theorem 1.

Lemma 6. *If X_1 and X_2 are logically closed, then*

$$(X_1 \otimes_1 F_2(G)) \cap (X_2 \otimes_2 F_1(G)) \subseteq F_1(G) + F_2(G).$$

Proof. et

$\Phi'_1 = X_1^{\leq \pi^1_{max}}$, where $\pi^1_{max} = \max\{k : X_1^{\leq k} \cup \Phi_2 \cup (X_1 \cap X_2) \text{ is consistent}\}$ and

$\Phi'_2 = X_2^{\leq \pi^2_{max}}$, where $\pi^2_{max} = \max\{k : \Phi_1 \cup X_2^{\leq k} \cup (X_1 \cap X_2) \text{ is consistent}\}$,
where (Φ_1, Φ_2) is the core of G.

Note that in the cases when π^i_{max} does not exist, we simply assume that it equals to $+\infty$. We claim that $X_1 \otimes_1 F_2(G) = \Phi'_1 + F_2(G)$ and $X_2 \otimes_2 F_1(G) = \Phi'_2 + F_1(G)$. We shall provide the proof of the first statement. The second one is similar.

Firstly, according to Lemma 2, $\Phi_1 \subseteq \Phi_1'$. Secondly, by Lemma 5, we have $X_1 \otimes_1 F_2(G) = X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$. Therefore to show $X_1 \otimes_1 F_2(G) = \Phi_1' + F_2(G)$, we only need to prove that $X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) = \Phi_1' + \Phi_2 + (X_1 \cap X_2)$. This is because $\Phi_2 + (X_1 \cap X_2) \subseteq F_2(G)$, $F_2(G) \subseteq \Phi_1 + \Phi_2 + (X_1 \cap X_2)$ and $\Phi_1 \subseteq \Phi_1'$. By the construction of prioritized revision, we can easily verify that $\Phi_1' + \Phi_2 + (X_1 \cap X_2) \subseteq X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$. Therefore we only have to show the other direction, i.e., $X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) \subseteq \Phi_1' + \Phi_2 + (X_1 \cap X_2)$.

If $\Phi_1' = X_1$, then $X_1 \cup (\Phi_2 + (X_1 \cap X_2))$ is consistent. It follows that $X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) \subseteq X_1 + (\Phi_2 + (X_1 \cap X_2)) = \Phi_1' + \Phi_2 + (X_1 \cap X_2)$, as desired. If $\Phi_1' \neq X_1$, according to the definition of π^1_{max}, we have $X_1^{\leq \pi^1_{max}+1} \cup \Phi_2 \cup (X_1 \cap X_2)$ is inconsistent. Therefore there exists $\psi \in X_1^{\leq \pi^1_{max}+1}$ such that $\neg\psi \in \Phi_2 + (X_1 \cap X_2)$. Now we assume that $\varphi \in X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$. If $\varphi \notin \Phi_1' + \Phi_2 + (X_1 \cap X_2)$, then $\{\neg\varphi\} \cup \Phi_1' \cup \Phi_2 \cup (X_1 \cap X_2)$ is consistent. So is $\{\neg\varphi \vee \psi\} \cup \Phi_1' \cup \Phi_2 \cup (X_1 \cap X_2)$. Notice that $\neg\varphi \vee \psi \in X_1^{\leq \pi^1_{max}+1}$. There exists $H \in X_1 \Downarrow (\Phi_2 + (X_1 \cap X_2))$ such that $\{\neg\varphi \vee \psi\} \cup \Phi_1' \subseteq H$. Since $\varphi \in X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2))$ and H is logically closed, we have $\psi \in H$, which contradicts the consistency of $H \cup (\Phi_2 + (X_1 \cap X_2))$. Therefore $X_1 \otimes_1 (\Phi_2 + (X_1 \cap X_2)) \subseteq \Phi_1' + \Phi_2 + (X_1 \cap X_2)$.

Finally we prove the claim of the lemma. Let $\varphi \in (X_1 \otimes_1 F_2(G)) \cap (X_2 \otimes_2 F_1(G))$. We then have $\varphi \in (\Phi_1' + F_2(G)) \cap (\Phi_1' + F_2(G))$. For $\varphi \in \Phi_1' + F_2(G)$, there exists a sentence ψ_2 such that $F_2(G) \vdash \psi_2$ and $\varphi \vee \neg\psi_2 \in \Phi_1'$. Similarly, there exists a sentence ψ_1 such that $F_1(G) \vdash \psi_1$ and $\varphi \vee \neg\psi_1 \in \Phi_2'$. It turns out that $\varphi \vee \neg\psi_1 \vee \neg\psi_2 \in \Phi_1' \cap \Phi_2'$. Thus $\varphi \vee \neg\psi_1 \vee \neg\psi_2 \in X_1 \cap X_2$. However, $X_1 \cap X_2 \subseteq F_1(G) + F_2(G)$. It follows that $\varphi \vee \neg\psi_1 \vee \neg\psi_2 \in F_1(G) + F_2(G)$. Note that $\psi_1 \wedge \psi_2 \in F_1(G) + F_2(G)$. Therefore we conclude that $\varphi \in F_1(G) + F_2(G)$.

4 Conclusion and Related Work

We have presented a logic-based bargaining solution based on Zhang and Zhang's model [12]. We have shown that the solution satisfies the fixed-point property, which asserts that the procedure of negotiation can be viewed as a course of mutual belief revision. The result is interesting not only because the result itself presents a desirable logical property of bargaining solutions but also establishes a link between bargaining and multi-agent belief revision. On the one hand, efforts have been made to the investigation of multi-agent belief revision [15,16], the research is far from satisfaction. On the other hand, bargaining have been a research topic in game theory for a few decades with sophisticated theory and variety of applications. It is easy to see that all the concepts introduced in this paper for the two-player bargaining game can be easily extended to the n-player cases. However, the extension of fixed-point property of mutual belief revision can be extremely hard. Therefore the link between bargaining and belief revision could give us a better understanding of multi-agent belief revision and could give us some hints towards the research.

The fixed-point property for negotiation functions was proposed by Zhang et al. [11]. However, there was no concrete negotiation function is constructed to satisfy the property. Meyer *et al.* gave a construction of negotiation function based on belief revision and discussed their logical properties [9,10]. Zhang and Zhang presented another belief-revision-based bargaining solution [12,13], which is similar to ours. However it is not too hard to verify that none of the above mentioned solutions satisfies the fixed-point property. Jin *et al.* [17] presents a mutual belief revision function that satisfies a fixed-point condition. However, the construction of the function is defined on belief revision operator and the fixed-point condition describes totally different property, which says that mutual belief revision is closed under iteration.

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