

Mining for Statistical Models of Availability in Large-Scale Distributed Systems: An Empirical Study of SETI@home

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- P2P, Grid, Cloud, and Volunteer computing systems

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Main Motivation

Effective Resource Selection for Stochastic Scheduling Algorithms

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Effective Resource Selection for Stochastic Scheduling Algorithms

Goal

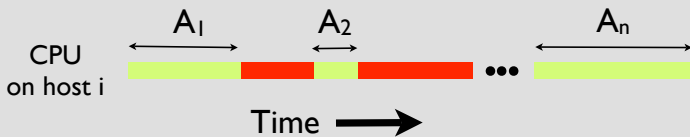
Model of host availability
(i.e., subset of hosts with the same availability distribution)

Outline

- 1 Introduction and Motivation
- 2 Measurement
 - Remove outliers
- 3 Modelling Process
 - Randomness Tests
 - Clustering
 - Model fitting
- 4 Discussions
 - Significance of Clustering Criteria
 - Scheduling Implications
- 5 Related Work
- 6 Conclusion and Future Work

Define Availability

CPU availability on each host



Length of Availability Intervals: A_1, A_2, \dots, A_n

Measurement Method



BOINC

- Middleware for volunteer computing systems
- Underlying software infrastructure for projects such as SETI@home

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BOINC

- Middleware for volunteer computing systems
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We instrumented the BOINC client to collect CPU availability traces:

- Total number of host traces: 226,208
- Collection period: April 1, 2007 - Jan 1, 2009
- Total CPU time: 57,800 years
- Number of intervals: 102,416,434
- Assume 100% or 0% availability



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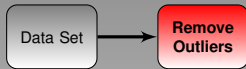
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Outliers



Outliers

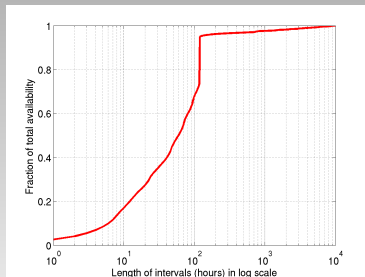


Check for outliers: Artifacts resulted from a benchmark run periodically every five days

Outliers



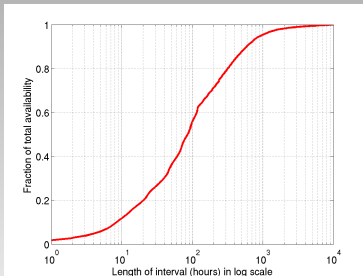
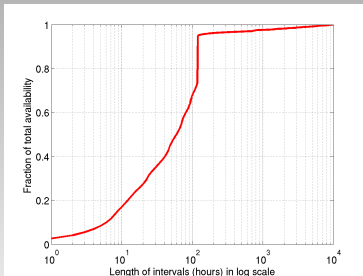
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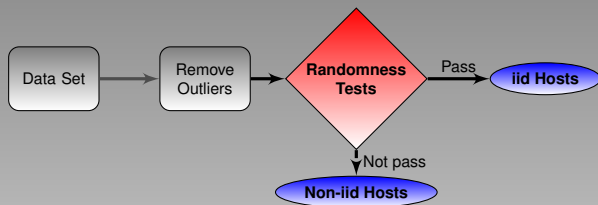
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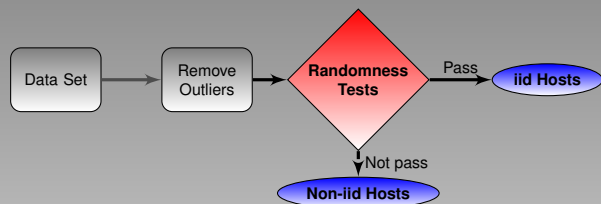
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Randomness Tests



To determine which hosts have truly random availability intervals

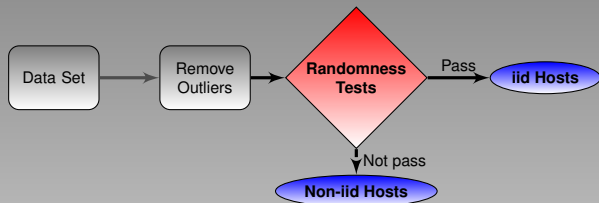
Randomness Tests



To determine which hosts have truly random availability intervals
Four well-known non-parametric tests:

- Runs test
- Runs up/down test
- Mann-Kendall test
- Autocorrelation function test (ACF)

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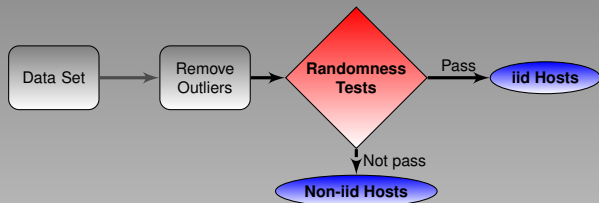


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Test	Runs std	Runs up/down	ACF	Kendall	All
# of hosts	101649	144656	109138	101462	57757
Fraction	0.602	0.857	0.647	0.601	0.342

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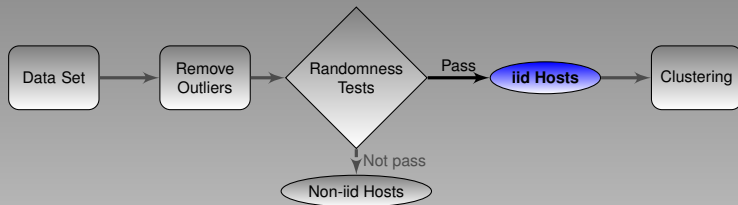
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Result: 34% are i.i.d. hosts (2.2 PetaFLOPS)

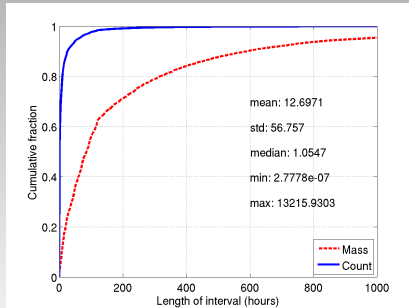
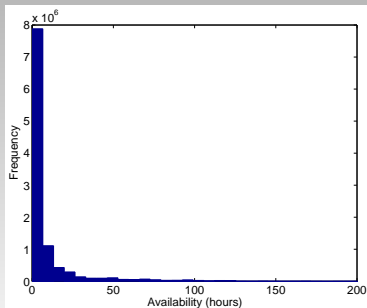
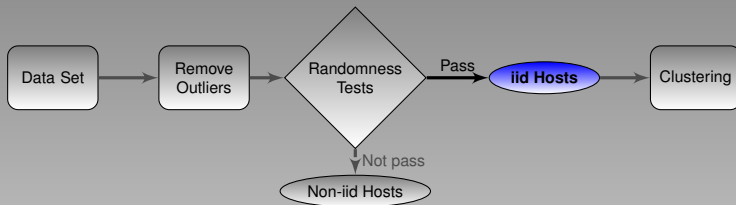
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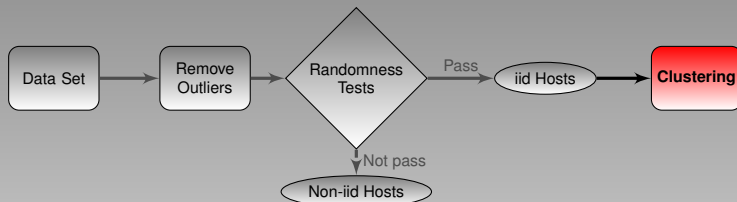
Distribution of Availability Intervals



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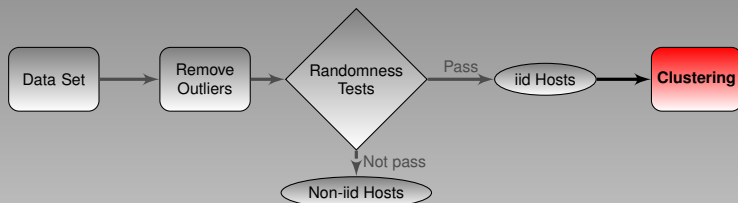


Clustering Method



Generate a few clusters based on availability distribution function

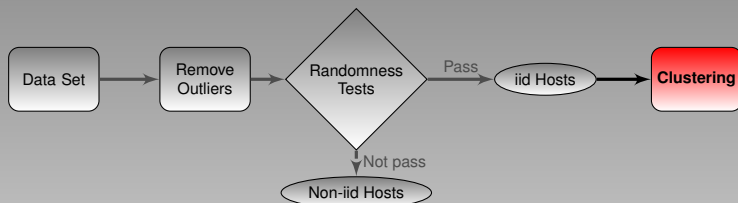
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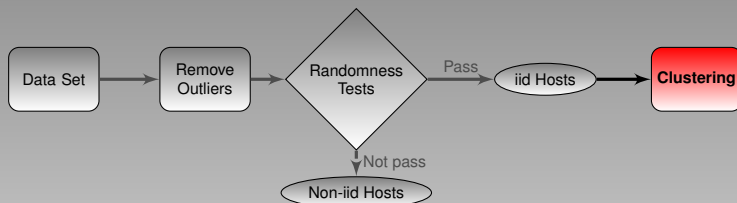
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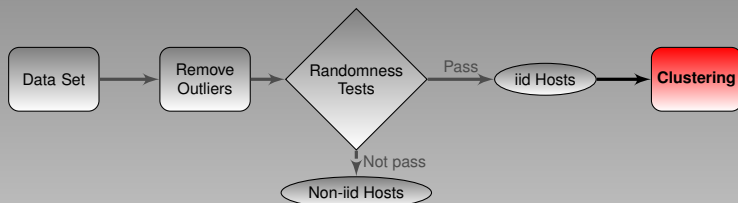
Clustering Method



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Method:

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 - Compute all permutations
 - Memory intensive

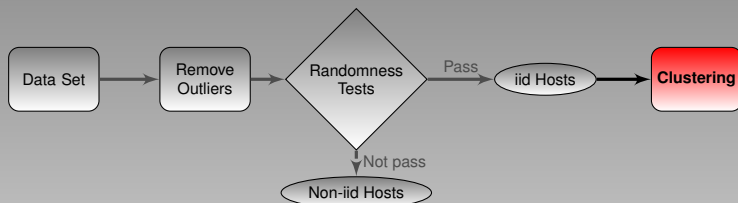
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Method:

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- K-means (fast K-means)

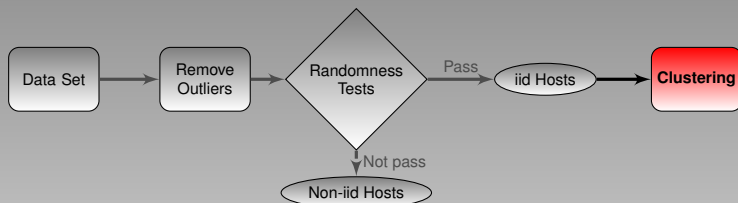
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Clustering Method

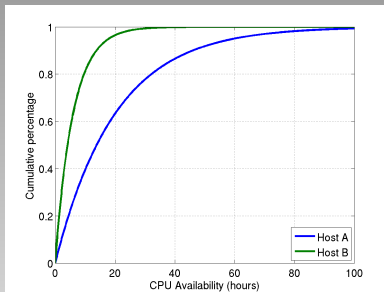


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 - Fast convergence
 - Dependent on initial centroids

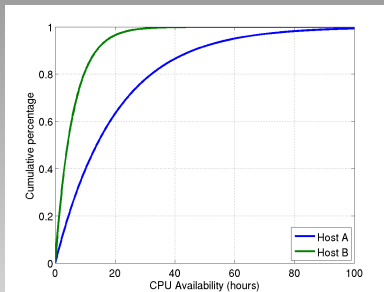
Distance Metrics

Distance between CDF of two hosts



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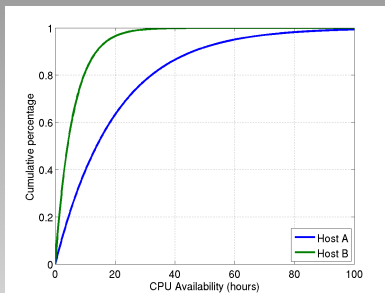
Distance between CDF of two hosts



- Kolmogorov-Smirnov: Maximum difference between two CDFs

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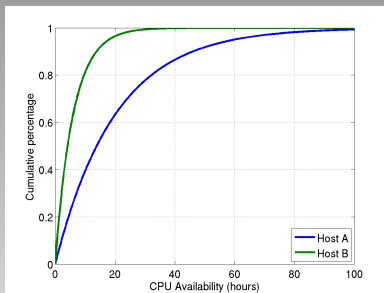
Distance between CDF of two hosts



- Kolmogorov-Smirnov: Maximum difference between two CDFs
- Kuiper: Maximum deviation above and below of two CDFs

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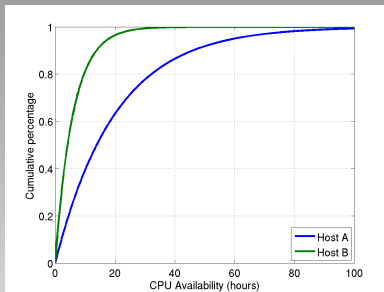
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- Kolmogorov-Smirnov: Maximum difference between two CDFs
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- Anderson-Darling: Area between two CDFs, more weight on the tail

Distance Metrics

Important Challenge:

Number of samples in each CDF

- Few samples → not enough confidence on the result

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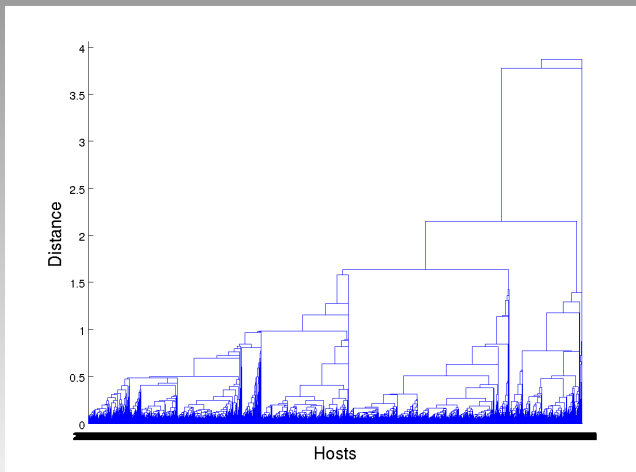
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Number of samples in each CDF

- Few samples → not enough confidence on the result
- Too much samples → the metric will be too sensitive
- **Data Set**: different hosts have different number of samples
- **Our solution**: randomly select a fixed number of intervals from each host (i.e., 30 samples)

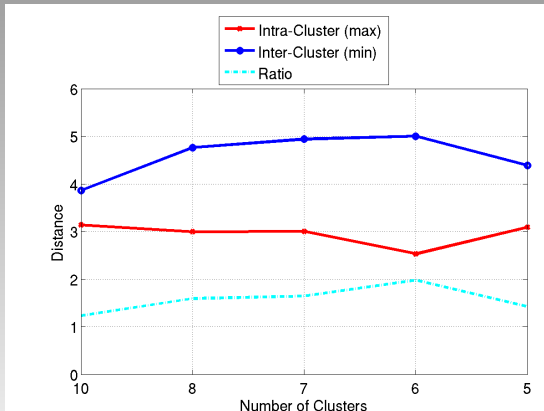
Clustering Results

Dendrogram of hierarchical clustering: 5-10 distinct groups (bootstrap)

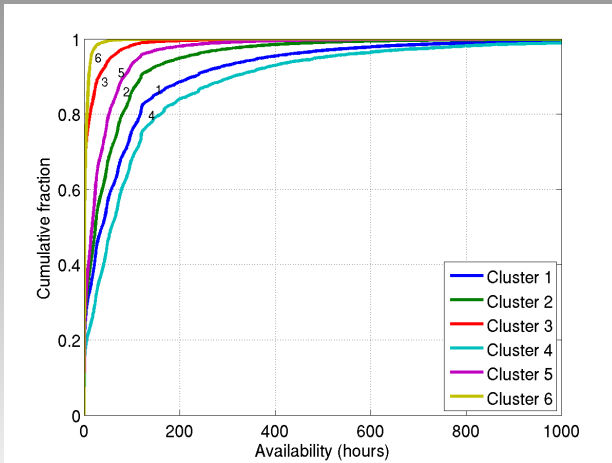


Clustering Results

Comparison of distances in clusters (k-means for all iid hosts):



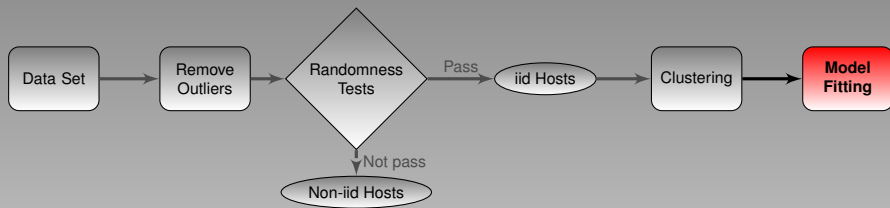
EDF of clusters



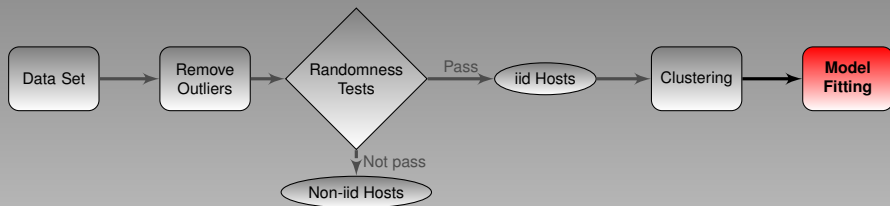
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Methods



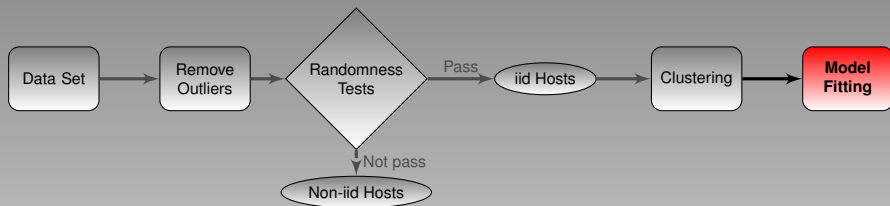
Methods



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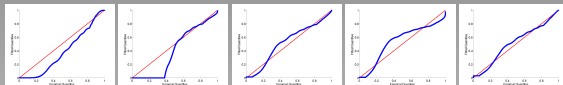
Target Distributions:

- Exponential
- Pareto
- Weibull
- Log-normal
- Gamma

Graphical Test

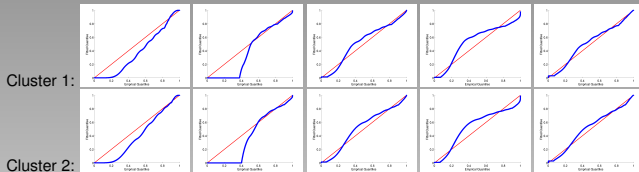
PP-plots: Exponential, Pareto, Weibull, Log-normal, Gamma

Cluster 1:



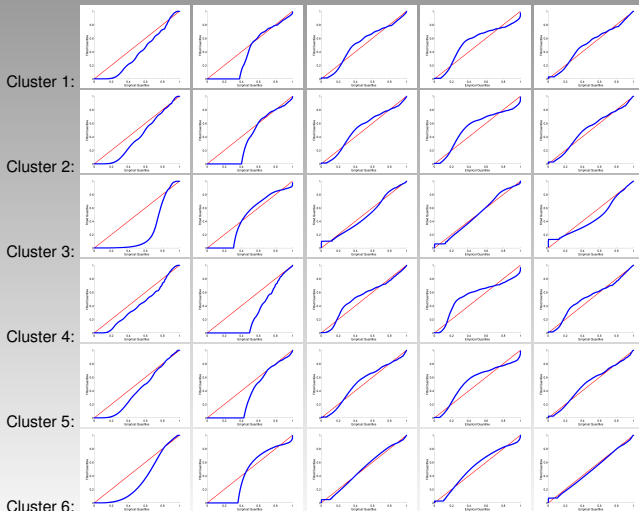
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Goodness Of Fit Tests

Generate p-values by two GOF tests (average over 1000 runs):

- Kolmogorov-Smirnov (KS) test
- Anderson-Darling (AD) test

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Data sets	Exponential		Pareto		Weibull		Log-Normal		Gamma	
	AD	KS	AD	KS	AD	KS	AD	KS	AD	KS
All iid hosts	0.004	0.000	0.061	0.013	0.581	0.494	0.568	0.397	0.431	0.359
Cluster 1	0.155	0.071	0.029	0.008	0.466	0.243	0.275	0.116	0.548	0.336
Cluster 2	0.188	0.091	0.020	0.004	0.471	0.259	0.299	0.128	0.565	0.384
Cluster 3	0.002	0.000	0.068	0.023	0.485	0.380	0.556	0.409	0.372	0.241
Cluster 4	0.264	0.163	0.002	0.000	0.484	0.242	0.224	0.075	0.514	0.276
Cluster 5	0.204	0.098	0.013	0.002	0.498	0.296	0.314	0.153	0.563	0.389
Cluster 6	0.059	0.016	0.033	0.009	0.570	0.439	0.485	0.328	0.538	0.467

Some properties of clusters

Clusters	# of hosts	% of total avail.	mean (hrs)	Best fit	Parameters	
					<i>shape</i>	<i>scale</i>
All iid hosts	57757	1.0	12.697	Weibull	0.3787	3.0932
Cluster 1	3606	0.16	90.780	Gamma	0.3131	289.9017
Cluster 2	9321	0.35	54.563	Gamma	0.3372	161.8350
Cluster 3	13256	0.22	11.168	Log-Normal	-0.8937	3.2098
Cluster 4	275	0.01	123.263	Gamma	0.3739	329.6922
Cluster 5	1753	0.05	34.676	Gamma	0.3624	95.6827
Cluster 6	29546	0.20	4.138	Weibull	0.4651	1.8461

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- Decreasing hazard rate

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Significance of Clustering Criteria

Could the same clusters have been found using some other static criteria?

Significance of Clustering Criteria

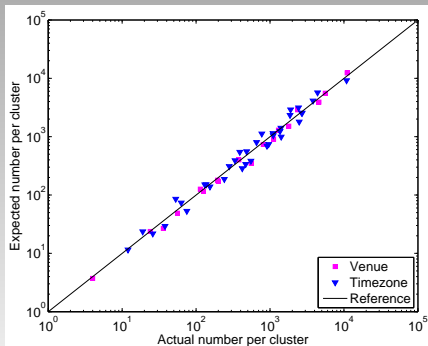
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- Cluster by Time zone: 6 different time zones

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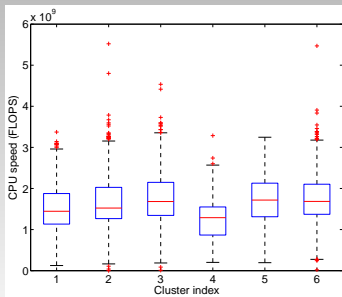
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- Cluster by venue: Work, Home, School
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- Cluster by CPU speed



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Scheduling Implications

Scheduling accuracy

Global model vs. Individual cluster model

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Global model vs. Individual cluster model

Ex: Completion probability of a 24-hour task:

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Scheduling accuracy

Global model vs. Individual cluster model

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- Single job: Prediction of task failure

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Resource Selection/Replication

- Single job: Prediction of task failure
- Multi-job: How the task size distribution follows the availability distribution

Related Work

Different from other research

- Measurement

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 - Resource type: home, work, and school

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Related Work

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- Measurement
 - Resource type: home, work, and school
 - Scale: 200,000 hosts
 - Duration: 1.5 years
 - Availability : CPU availability
- Modelling
 - Classification according to randomness tests
 - Cluster-based Model vs Global Model

Conclusion and Future Work

Discovering availability models for host subsets from a distributed system

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- Methodology

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 - Partitioning hosts into subsets by their availability distribution

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 - Partitioning hosts into subsets by their availability distribution
- Modelling (Apply the methodology for the SETI@home)
 - 34% of hosts have truly random availability intervals

Conclusion and Future Work

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Conclusion

- Methodology
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 - Classification based on the randomness tests (iid vs non-iid)
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Future Work

- Apply the result for improving makespan of DAG-applications
- Explore ability of clustering dynamically while the system is on-line

Failure Trace Archive

<http://fta.inria.fr>

- Repository of availability traces of parallel and distributed systems, and tools for analysis
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More Details

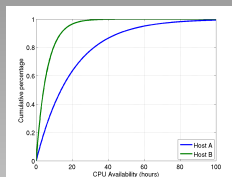
- Poster Session at MASCOTS 2009 (Today 19:00-21:00)
- Website: <http://fta.inria.fr>

Thank You

Questions?

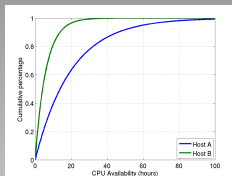
Distance Metrics

Distance between CDF of two hosts



Distance Metrics

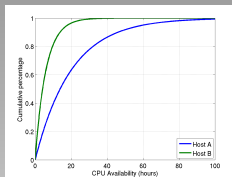
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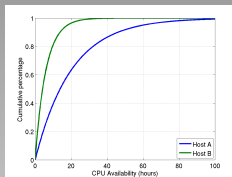
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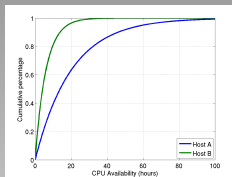


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- Anderson-Darling: $Q_n = \int_{-\infty}^{\infty} [F(x) - F_n(x)]^2 \psi(F(x)) dF$

$$\psi(F(x)) = \frac{1}{F(x)(1-F(x))}$$

Fitting with Hyper-Exponential

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- Expectation Maximization (EM) [using EMpht package]
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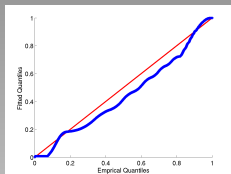
We used MM for 2-phase hyper-exponential by the first two moments as follows:

$$p = \frac{1}{2} \left(1 - \sqrt{\frac{CV^2 - 1}{CV^2 + 1}} \right)$$

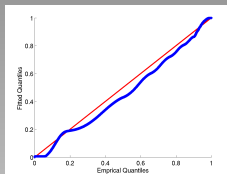
$$\lambda_1 = \frac{2p}{\mu}$$

$$\lambda_2 = \frac{2(1-p)}{\mu}$$

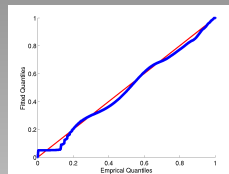
PP-Plots



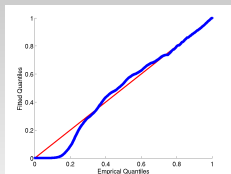
(a) Cluster 1



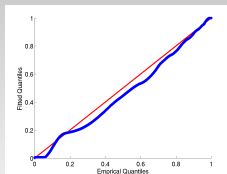
(b) Cluster 2



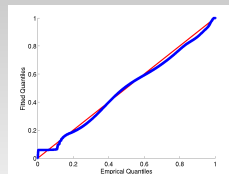
(c) Cluster 3



(d) Cluster 4



(e) Cluster 5



(f) Cluster 6

Goodness of Fit Tests

Data sets	Hyper-Exponential (MM)			Hyper-Exponential (EM)		
	Parameters	AD	KS	Parameters	AD	KS
All iid hosts	$\rho_1 = 0.024 \lambda_1 = 0.004$ $\rho_2 = 0.976 \lambda_2 = 0.154$	0.026	0.005	$\rho_1 = 0.197 \lambda_1 = 0.0179$ $\rho_2 = 0.279 \lambda_2 = 29.171$ $\rho_3 = 0.524 \lambda_3 = 0.316$	0.531	0.375
Cluster 1	$\rho_1 = 0.115 \lambda_1 = 0.003$ $\rho_2 = 0.885 \lambda_2 = 0.019$	0.287	0.119	$\rho_1 = 0.180 \lambda_1 = 14.401$ $\rho_2 = 0.820 \lambda_2 = 0.009$	0.450	0.318
Cluster 2	$\rho_1 = 0.114 \lambda_1 = 0.004$ $\rho_2 = 0.886 \lambda_2 = 0.032$	0.275	0.113	$\rho_1 = 0.183 \lambda_1 = 12.338$ $\rho_2 = 0.817 \lambda_2 = 0.015$	0.512	0.403
Cluster 3	$\rho_1 = 0.030 \lambda_1 = 0.005$ $\rho_2 = 0.970 \lambda_2 = 0.174$	0.005	0.000	$\rho_1 = 0.341 \lambda_1 = 0.031$ $\rho_2 = 0.261 \lambda_2 = 71.852$ $\rho_3 = 0.398 \lambda_3 = 1.923$	0.561	0.434
Cluster 4	$\rho_1 = 0.136 \lambda_1 = 0.002$ $\rho_2 = 0.864 \lambda_2 = 0.014$	0.448	0.273	$\rho_1 = 0.694 \lambda_1 = 0.020$ $\rho_2 = 0.306 \lambda_2 = 0.003$	0.473	0.274
Cluster 5	$\rho_1 = 0.105 \lambda_1 = 0.006$ $\rho_1 = 0.895 \lambda_2 = 0.052$	0.295	0.122	$\rho_1 = 0.173 \lambda_1 = 13.374$ $\rho_2 = 0.827 \lambda_2 = 0.024$	0.523	0.393
Cluster 6	$\rho_1 = 0.010 \lambda_1 = 0.005$ $\rho_2 = 0.990 \lambda_2 = 0.478$	0.114	0.038	$\rho_1 = 0.516 \lambda_1 = 0.131$ $\rho_2 = 0.150 \lambda_2 = 163.771$ $\rho_3 = 0.334 \lambda_3 = 2.411$	0.572	0.470